SIMULTANEOUS PRICING AND LOT SIZING FOR A DETERIORATING PROCESS

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ABSTRACT

This paper develops Banerjee's model (2005), on concurrent pricing and lot sizing for a supplier on the bases of contractual agreement with a buyer, by considering condition that the supplier's production system deteriorates over time/usage. Further, it is also assumed that the bargaining position of the buyer is stronger than that of the supplier so its economic lot size policy is used to determine the supplier's production batch size (i.e. the supplier production batch size is an integer multiple of the buyer's order quantity). The objective of the supplier here is to determine the product's selling price, in conjunction with an appropriate lot size policy, to maximize targeted gross profit per unit. Any relevant costs considered by the supplier are setup cost, manufacturing cost, holding cost, restoration cost, and repairing cost. To solve the model, a simple algorithm is employed, and a numerical example using Banerjee's parameter is implemented to illustrate the works of the model.

Keywords: pricing, lot sizing, deteriorate, Banerjee's model

Introduction

Determining optimal production lot size is a problem always faced by a supplier in running its production policy to supply a product to its customer. The main objective to be achieved from that activity is to minimize the total production cost. The classical Economic Manufacturing Quantity (EMQ) model (see for example Silver et al., 1998) is often employed to solve the problem because of its simplicity. However, this model implicitly assumes that the production process is perfect, whereas in the real situation its performance will inevitably deteriorate due to usage or age.

Deterioration may shift the production state from in control to out-of control state within production period and need to be restored at the end of the production cycle. Thus, restoration cost should directly be added to the production cost. Porteus (1986) and Rossenblatt and Lee (1986) initially studied the effect of process deterioration on the optimal EMQ. Rossenblatt and Lee considered the shift of the state is exponentially distributed, while Porteus uses a given probability each time an item produced. Both of the researches have the same conclusion that the optimal lot size for imperfect production system is smaller than the optimal EMQ.

Kim and Ha (2003) studied the lot sizing policy in the frame of joint coordination between a buyer and a vendor. Both production and delivery lot size were determined simultaneously in achieving the best solution for both parties. Here, the supplier production batch size is an integer multiple of the buyer's order quantity. However, all papers mentioned above focus on determining production lot size to minimize total production cost using given product price. If the supplier produces on make-to-order environment and on a contractual based, it is very important to include pricing on the model.

The common issue to discuss between a supplier and a buyer in the contract negotiation is the product's selling price. A supplier has to pay attention on this matter carefully since it is a very sensitive case for a buyer in dealing a contract. Overpricing case will encourage a buyer to seek another source of supply while under pricing case will shorten the supplier's profit that is certainly an undesired condition for a supplier.

The product's selling price will be used to determine the buyer's purchasing policy while for the supplier the buyer's purchasing policy will influence the production/ inventory policy and finally will directly affect to the product's selling price. From this case, the price and the production policy need to be determined concurrently.

Banerjee (1986) conducted a research on product pricing for supplier that produces on the bases of contractual agreement. It is assumed that the supplier production lot size equals to transferring lot size. In the following paper, Banerjee (2005), this assumption was relaxed by considering production lot size is a multiple integer of the delivery lot size. Both papers deal with pricing and lot sizing for a supplier concurrently by assuming that the customer is rational, i.e. it follows its own optimal purchasing policy. The objective of the research is to determine the product's selling price, in conjunction with an appropriate production/ inventory policy, so that a given profit level is achieved. However, both papers implicitly assume that the production process is perfect while, in real condition, the supplier's production system will deteriorates due to usage.

The purpose of this paper is to determine the appropriate price, in conjunction with appropriate lot size policy, for a supplier that produces and supplies a product to a buyer on the bases of contractual agreement. It is considered that the supplier's production process deteriorates due to usage or age such as fatigue, corrosion, and wearing age. The production shifts from in-control into out of control with a given probability each time an item produced. It is assumed that the customer follows its optimal purchasing policy. Although the supplier has no exact information of buyer's relevant cost parameter values, it is sufficient for the supplier to have a good estimate of them. Here, the production lot size is restricted to be a multiple integer of buyer's ordering quantity.

The remainder of this paper is organized as follows. The notation and assumptions are given in Section 2 while the mathematical model is established in Section 3. Section 4 provides a numerical example. Finally, conclusion and implication of the results are summarized in Section 5.

Notation and assumptions

The notation here is adopted from Banerjee's model and summarized as follows:

D : the demand of the product on the buyer per unit time
P : the supplier's production rate per unit of time
S_1 : the buyer's ordering cost per order ($/ order)
S_2 : the supplier's production setup cost per setup ($/ setup)
h_1 : the buyer's inventory holding cost rate in ($/ time unit)
h_2 : the supplier's inventory holding cost rate in ($/ time unit)
C_1 : the supplier's selling price per unit or the customer's unit purchase cost
C_2 : the supplier's production cost
TRC_1 : the customer's relevant cost per period
TRC_2 : the supplier's total relevant cost per period
G : the supplier's gross margin (profit) goal in $ per unit
Q : the buyer's order quantity in unit
K : the supplier's production batch size multiplier
θ : the proportion of non-conforming item produced during the out-of control state
q : the probability of the system stay in the in-control state when producing an item
η : The restoration cost per restoration ($/restoration)
c_r : The repairing cost per unit ($/unit)

The following assumptions are used during model formulation:

1. The product is repairable and produced by a single production system
2. The system will stay in the in-control state with probability q, and shift to out-of control state with probability 1-q, where $0<q<1$, during the production of an item.
3. No stock out are allowed
4. The customer's ordering behavior is guided by its economic purchasing policy

Model Formulation

The buyer's economic purchasing policy is the ordering quantity, Q, which minimizes its total relevant cost. The buyer's relevant costs considered are purchasing, ordering, and holding cost. Hence, the total customer's relevant cost per period is the sum of them and can be expressed as

$$TRC_1 = C_1 D + S_1 D / Q + 0.5 h_1 C_1 Q \quad ...(1)$$

Taking the first derivation of the equation and set it equals to zero (see Silver et al., 1998) we easily find the customer's economic ordering quantity (*EOQ*) as follow

$$Q^* = \sqrt{2 S_1 D / h_1 C_1} \quad ...(2)$$

We know that the supplier has no exact information of both the parameter value of holding cost (h_1) and ordering cost (S_1), however it is sufficient for the supplier to have a good estimation of them. From equation (2), the supplier is able to estimate the buyer's economic ordering quantity (Q^*).

Based on the supplier's perspective, the total supplier's relevant cost per period is the sum of setup cost, production cost, holding cost, restoration cost, and repairing cost. For the sum of setup cost, production cost, and holding cost, we adopt Banerjee's (2005) model and written as

$$= C_2 D + D S_2 / KQ + 0.5 h_2 C_2 Q \left((2-K) D / P + K - 1 \right) \quad ...(3)$$

The expectation of total restoration cost per period is the multiplying of the restoration cost per restoration, the number of setup needed per period, and the probability of the production state to shift to out-of control state within production cycle. The production process may stay in the in-control state with probability q each time an item produced. If the supplier's production lot size KQ, where Q is the buyer's economic order quantity and K is the positive integer multiplier. If m is the number of item produced during in-control state, the probability of producing m can be written below

$$P\{M=m\}=\begin{cases} q^{m}(1-q) & m=0,1,\ldots,KQ-1 \\ q^{KQ} & m=KQ \end{cases} \quad \text{...(4)}$$

From equation (4) we find that the probability of the system will stay in the in-control state equals to q^{KQ}. Hence, the expectation of total restoration cost can be expressed as

$$=\eta.D\left(1-q^{KQ}\right)/KQ \quad \text{...(5)}$$

The expected number of item produced when the system is in in-control, $E[M]$ is

$$E[M]=q(1-q^{KQ})/(1-q) \quad \text{...(6)}$$

Thus, we can write $KQ\text{-}E[M]$ as the expectation of the number of item produced during out-of control state for lot size KQ. The end products produced by the production system are divided into 2 categories, namely conforming item and non-conforming item. It is assumed that the production process will produce non-conforming item with proportion θ during out-of control state. Non conforming items will be detected during inspection and will be repaired with cost per unit, c_r. Then, the expectation of total repairing cost can be written below

$$=c_r\theta.D\left(1-q(1-q^{KQ})/KQ(1-q)\right) \quad \text{...(7)}$$

Then, the total supplier's relevant cost is the sum of equation (3), (5), and (7) and the result is

$$TRC_2(K,Q)=D(S_2/KQ+C_2+0.5h_2C_2Q((2-K)/P+ (K-1)/D)+\eta\left(1-q^{KQ}\right)/KQ+c_r\theta\left(1-q(1-q^{KQ})/KQ(1-q)\right)\Big) \quad \text{...(8)}$$

K^* that minimize $TRC_2(K,Q)$ exist if these inequalities, $TRC_2(K^*+1,Q)$ - $TRC_2(K^*,Q) \geq 0$ and $TRC_2(K^*-1,Q)$ - $TRC_2(K^*,Q) \geq 0$, are hold. If q is very close to zero, we can use approximation $q^{KQ}=1+KQ\ln q+(KQ\ln q)^2$ and we have

$$K^*(K^*-1)\leq S_2D/Q^2Z\leq K^*(K^*+1) \quad \text{...(9)}$$

Where $Z=D(0.5h_2C_2(1/D-1/P)-\eta\ln^2 q-c_r\theta q\ln^2 q/(1-q))$, replacing Q in equation (9) with the buyer's economic ordering quantity resulting from equation (1) we have

$$K^*(K^*-1)\leq S_2C_1/2.\rho.Z\leq K^*(K^*+1) \quad \text{...(10)}$$

where ρ is the buyer's ordering cost to holding cost ratio.

From equation (10) we know that since the variable C_1 is unknown we can not find K^*. K^* and C_1^* have to solve concurrently. Here, the supplier is targeted to achieve a given gross profit per unit of G (note that the gross profit is the revenue subtracted by the supplier's relevant cost). Assuming the buyer uses its economical purchasing policy, we can replace Q in equation (8) with Q^* in equation (1) obtaining function $TRC_2(\mathrm{K},\mathrm{C}_1)$, and then the supplier's gross profit per unit is calculated as follow

$$G = \left[\begin{array}{l} C_1 - C_2 - S_2\sqrt{C_1}\,/\,K\alpha - 0.5h_2C_2\alpha\big((2-K)/P + (K-1)/D\big)/\sqrt{C_1} - \\ cr.\theta - \big(\eta - cr.\theta q/(1-q)\big)\sqrt{C_1}\left(\dfrac{1-q^{K\alpha/\sqrt{C_1}}}{K\alpha}\right) \end{array}\right] \quad ...(11)$$

where $\alpha = \sqrt{2S_1D/h_1}$

Equation (11) has two unknown variables, namely an integer value K and the real value C_1, that have to be solved simultaneously. We use simple algorithm proposed by Banerjee to find K^* and C_1^* written as follows

Step 1 : Initialize $C_1=C_2+G$

Step 2 : Find K^* using condition (10) and current value of C_1

Step 3 : Substitute the value of K^* found in step (2) into equation (11) and find new C_1.

Step 4 : Go to step 2 and recalculate K^* using C_1 obtained from step 3. If old and new value of K^* is the same then stop, otherwise repeat step 2 and 3 until convergence is obtained.

Numerical Example

Banerjee's data are adopted for illustrating the model and algorithm developed above. They are as follows:

D = 10,000 unit/ year
P = 20,000 unit/ year
S_1 = \$20 per order
S_2 = \$200 per setup
C_2 = \$ 5 per unit
G = \$1.5 per unit
$h_1 = h_2$ = \$0.2/unit/year

We use algorithm outlined earlier to find K^* and C_1^*.

STEP 1 : $C_1= 5 + 1.5 = 6.5$

STEP 2 : Using $C_1 = 6.5$ for solving equation (9) resulting $K^* = 4$.

STEP 3 : Substituting $K^* = 4$ in (10) and solve it, then we obtain $C_1= 6.9948$

STEP 4 : Repeating step 2 using value C_1=6.9948, we get K^*=4. We terminate the iteration since there is no change in the value of K^*.

The conclusion for the numerical example above is the supplier's selling price \$6.9948 with four time equal-batch delivery. Further, we only need one iteration to obtain the values K^* and C_1^*. It shows that the algorithm works efficiently.

Findings and conclusions

From modeling formulation and numerical example presented earlier, there are some interesting points to draw. They are:

1. Since the last two elements of Z are always positive, it will result K^* that smaller than that of in Banerjee's or will be equal if $\eta/c_r.\theta=q/(1-q)$.
2. Two cost elements added (restoration cost and repairing cost) cause the supplier's product price higher than that of in Banerjee's.
3. From (9), a supplier will get benefit from delivering order in smaller batch instead of in lot-for-lot strategy if $S_2C_1/2.\rho.Z > 2$

This paper presents a model to determine the supplier's product price and the number of deliveries (lot sizing policy) concurrently. It extends Banerjee's model by accommodating condition of the supplier's production system which deteriorates due to usage. However, all parameters used in this paper are still assumed to be deterministic. Relaxing this assumption would be the next direction for further research.

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