

## Understanding High School Students' Errors in Solving Mathematics Problems: A Phenomenological Research

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DOI: 10.23917/ijolae.v7i1.24005

Received: September 5<sup>th</sup>, 2024. Revision: October 27<sup>th</sup>, 2024. Accepted: November 5<sup>th</sup>, 2024.

Available Online: November 26<sup>th</sup>, 2024. Publish Regularly: January, 2025.

### Abstract

This research aims to understand a phenomenon regarding high school students' errors in solving mathematics problems using a qualitative approach with phenomenology as the analysis framework. Data were collected through tests, classroom observations, documentation (students' answer sheets, list of attendees, and students' score lists), and unstructured phenomenological interviews with four purposively selected participants who met the selection criteria. The researchers used the mathematical problem-solving (MPS) model by Rott-Specht-Knippling and Aguas' phenomenological data analysis steps using the NVIVO 12 software to analyze the students' MPS process and identify their errors and the factors contributing to these errors. Errors were predominantly found in problems solved without engaging in the exploration phase. Analysis errors were the most common, while errors due to carelessness were the rarest. Factors contributing to these errors were identified across five domains: MPS Ability (MPSA), cognition, affection, motivation, and self-awareness. This research provides valuable insights into student errors in MPS for researchers and educators, particularly teachers, and provides recommendations for mathematics education policies and future research.

**Keywords:** cognition domain, educational intervention, education policies, educational standards, learning engagement, mathematical problem solving, phenomenology research, technology-assisted learning model

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## 1. Introduction

The primary goal of learning mathematics is to equip students with mathematical abilities. The mathematics learning process and the student's subsequent understanding of the subject are critical due to its significant impact on their future. The National Council of Teachers of Mathematics or NCTM (2000) emphasized that mathematical abilities encompass various skills, with mathematical problem solving (MPS) being a key component. Similarly, the Educational Standards,

Curriculum, and Assessment Agency or BSKAP (2022) specified that developing MPS abilities is a fundamental objective of mathematics education. Moreover, MPS abilities are considered essential to students' overall mathematical proficiency, as outlined in the pentagon framework of the mathematics curriculum in Singapore (Leong et al., 2011).

MPS is a crucial component of mathematics education (NCTM, 2000). It aims to enhance students' mathematical thinking

skills (Rott et al., 2021). The skill helps students acquire new mathematical knowledge, utilize strategies in various situations, and evaluate problem-solving methods (NCTM, 2000). Therefore, students need to master these intricate abilities (Kingsdorf & Krawec, 2014).

Pólya (2004) defines MPS as a problem-solving process that involves understanding the problem, designing a plan, implementing the plan, and reflecting on the solution. Schoenfeld (1985) emphasized that MPS deals with situations lacking a straightforward answer, requiring analysis, design, exploration, implementation, and verification. Solso (1995) also outlined MPS steps as problem identification, problem representation, planning the solution, executing the plan, evaluating the plan, and assessing the solution. Moreover, Eggen and Kauchak (1996) further broke down MPS into problem identification, problem formulation, strategy selection, strategy implementation, and results evaluation.

These definitions described the MPS model normatively as it refers to the idealization process. Rott et al. (2021) developed a descriptive MPS model that includes an analysis phase, an exploration phase, a planning phase, an implementing phase (sometimes combined as a planning-implementing phase), and a verification phase. Their MPS model is an advancement of the Schoenfeld model, offering the advantage of distinguishing between linear transitions (similar to Pólya and Schoenfeld's MPS model) and non-linear transitions (where phases may not follow a specific order, or some phases may be skipped). It also allowed for comparing routine and non-routine processes, particularly in the exploration phase in non-routine processes. However, this model is currently limited to Geometry topics. In this study, we focus on utilizing the phases from the Rott-

Specht-Knipping model as distinct categories for MPS process analysis.

Furthermore, Philip categorized MPS based on inner and outer structures, while Neuhaus distinguished them based on intuitive/creative and logical models. Rott et al. (2021) emphasized that the inner structure involves cognitive processes, while the outer structure involves observable actions in chronological phases. The intuitive or creative model includes preparation, incubation, illumination, and verification phases, while the logical model comprises suggestion, intellectualization, guiding ideas and hypotheses, reasoning narrowly, and testing hypotheses through action (Rott et al., 2021). This study focuses on the outer structure of MPS and the logical type.

In mathematics learning, various activities demonstrate the achievement of learning objectives, particularly in assessing students' mastery of MPS abilities. One such activity involves students solving mathematics problems, presented as mathematical word problems or non-story problems (Novak & Tassell, 2017). Mathematical word problems and non-story problems are types of mathematics problems that involve one or multiple steps to solve.

Mathematical word problems are story-based problems that relate to real-life situations (Böswald & Schukajlow, 2023; Milazoni et al., 2022; Verschaffel et al., 2000, 2020). They require students to understand context, interpret information, assign numerical values based on the scenario, and perform calculations (Novak & Tassell, 2017). On the other hand, non-story problems do not include a narrative or context. In this study, students were presented with non-story mathematics problems focusing on circle analysis in the Mathematics Specialization subject.

When students tackle non-story mathematics problems, they may encounter

obstacles that lead to difficulties in solving them. These difficulties often result in errors in their answers (Hasan et al., 2019; Lestari et al., 2019). Radatz (1980) stated that student errors highlight difficulties and a lack of understanding of mathematical concepts and problems. Dwita and Retnawati (2022) further elaborated that these challenges can lead to errors in students' problem-solving processes. Thus, students' struggles with solving mathematics problems, particularly non-story problems, contribute to their errors when responding to these problems.

Student errors in solving mathematics problems can be detrimental if left unaddressed. If a student's error is not corrected, the error will continue to stick to them and contribute to their difficulties in understanding other mathematical concepts. Radatz (1980) noted that student errors can persist throughout their school years unless teachers intervene.

Therefore, interventions are required to help students solve mathematics problems. Radatz (1980) explained that student errors can be analyzed as they are typically identifiable and systematic. According to Svenson et al. (1983), detailed error diagnosis can effectively intervene or remediate the student's issues in specific areas. This type of diagnosis is called an error analysis.

Error analyses are closely related to the MPS model. Radatz (1979) stated that a cognitive model is essential for identifying the causes of errors in solving mathematical problems. In this study, the MPS model by Rott-Specht-Knipping was utilized to identify the errors and factors contributing to the errors.

Moreover, Kingsdorf and Krawec (2014) stated that error analysis is a valuable tool for assessing mathematics learning by analyzing students' errors when solving problems. Rofi'ah et al. (2019) further elaborated that error analysis provides valuable insights

that can assist educators, particularly teachers, in identifying the types of errors made and the underlying factors contributing to these errors.

Previous research has explored the various obstacles that students may encounter when answering non-story mathematics problems. Some descriptive studies have analyzed students' errors in solving math problems, such as Pomalato et al. (2020) and Son et al. (2019). Meanwhile, Veloo et al. (2015) identified different types of errors, and case studies like Díaz et al. (2020) and Priyani and Ekwati (2018) investigated the issue further. However, there is a lack of research that thoroughly investigates student errors in solving non-story mathematics problems as a phenomenon. Existing studies often describe errors without examining the students' thought processes or identifying the root causes of these errors.

Understanding the phenomenon that may occur when students make errors in solving non-story mathematics problems is crucial. Moustaka defined a phenomenon as something that emerges in consciousness (Yüksel & Yıldırım, 2015). Phenomenons can be observed through events, experiences, constructs, orientations, concepts, and situations in the world (Alexander, 1970; Howard, 1994; Kalkan & Dağlı, 2021).

By understanding a phenomenon, researchers can delve into the core of an individual's life experience (Yüksel & Yıldırım, 2015). Thus, this study's primary focus is understanding a phenomenon regarding students' errors in solving mathematics problems among high school students. An in-depth study about this phenomenon was conducted to explore and clarify the errors made by students in their MPS process, including the types of errors and factors contributing to them. Therefore, this study addresses the following questions: (1) What process do

students follow when solving non-story mathematics problems? (2) What errors do students typically make when responding to non-story mathematics problems? and (3) What factors contribute to students' errors when solving non-story mathematics problems?

## 2. Method

This research employed a qualitative approach with a phenomenological analysis framework. Phenomenology aims to uncover and describe the fundamental structure of psychological phenomena (Aagaard, 2017; Alexander, 1970). The psychological significance of a specific phenomenon is derived from the participants' raw data descriptions, distilled to their essence, which should encompass all potential ways of experiencing the phenomenon (Aagaard, 2017).

Phenomenological research is highly suitable for revealing and understanding the phenomena that occur (Kalkan & Dağlı, 2021; Lukman et al., 2021; Yazıcı & Fidan, 2020). Phenomenology aims to clarify a phenomenon (Vagle, 2009) and explore perceptions or experiences (Liu & Winder, 2014). Therefore, it is well-suited for investigating students' errors in solving non-story mathematics problems.

### a. General Background

The research began with a study that utilized a quasi-experimental approach to investigate the impact of a technology-assisted learning model on the mathematical problem-solving ability (MPSA) of class XI students at a high school in Bintan, Indonesia. The findings indicated a significant improvement in the students' MPSA due to the use of the technology-assisted model. However, none of the students fell into the high or very high MPSA category (Table 1). This finding suggested that the students may have faced challenges,

leading to errors in their response to the non-story mathematics problems provided.

### b. Participants

The participants were selected based on purposive sampling criteria, which included their readiness to participate (evidenced by retaining their answer sheets and active participation in the classroom), willingness to be interviewed, and the potential value of the information they could provide (Etikan, 2016). Four female participants who met these criteria were selected. There is no correlation between sample size and study quality in phenomenological research (Bartholomew et al., 2021). The participants were categorized based on their MPSA levels (Table 1).

Table 1. Students' MPSA Levels

Category	Value (x)	Total
Very high	$80 < x \leq 100$	0
High	$60 < x \leq 80$	0
Medium	$40 < x \leq 60$	3
Low	$20 < x \leq 40$	18
Very Low	$x \leq 20$	12

Table 1 shows that most students fall into the low MPSA category. However, from the initial participant selection, only four participants met the criteria. Table 2 describes the four participants' MPSA level.

Table 2. Description of Participants Based on MPSA Level

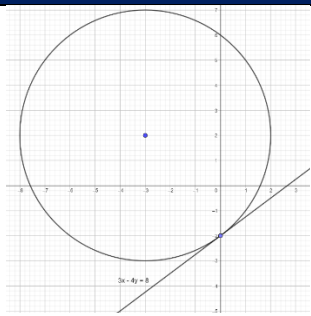
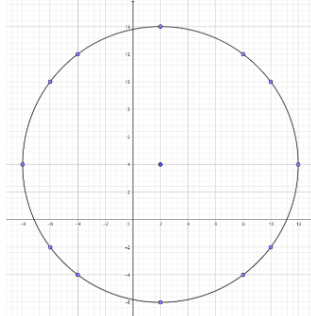
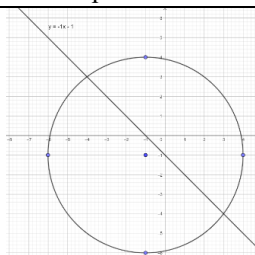
Code	x	MPSA Level
RS1	40	Low Category
RS2	53	Medium Category
RS3	59	Medium Category
RS4	26	Low Category

### c. Instruments and Procedures

The instrument in qualitative research is the researchers themselves (Wa-Mbaleka, 2020). However, other instruments were used to collect research data, such as the non-story

MPS test. This test presents open-ended problems to the students. Table 3 shows the non-story MPS test problems provided to the students.

**Table 3. Non-Story Mathematics Problems**

Item	Problem
1	 <p>Look at the graph. Determine the general form of the equation of a circle and its point of tangency to the line! (Note: the equation of the line is <math>3x - 4y = 8</math>)</p>
2	 <p>Look at the following circle graph. Determine the position of the line passing through points (4,0) and (0,2) on the circle!</p>
3	Determine the equation of the tangent line of a circle which has a tangent point (0,4) and whose center point is at $2x + 4y = 8$ , and the circle is tangent to the $x$ -axis and $y$ -axis.
4	 <p>Look at the following graph. Determine the equation of the tangent line to the circle where the tangent line is perpendicular to the line that intersects the circle. (Note: the equation of the line is <math>y = -x - 1</math>)</p>

The validity of this test was confirmed through the expert judgment of three experienced mathematics teachers with over 10 years of teaching experience each. The estimation of Aiken's V was 0.75 for every item, indicating moderate validity (Retnawati, 2016). Additionally, the test's validity was

supported by estimating reliability ( $\alpha$  and  $\omega$  coefficients) and conducting item analysis. In item analysis, discriminatory power (Pearson correlation/p) and level of difficulty (T) were estimated. Table 4 presents the item analysis results, reliability estimation, and the decision for the MPS test instrument.

**Table 4. Item Analysis Results, Reliability Estimation, and The Decision of the MPS Test instrument**

Item	p	$\alpha$	$\omega$	T	Decision
1	0.807	0.726	0.722	0.221 (Hard)	Item accepted
2	0.646			0.111 (Hard)	Item accepted
3	0.825			0.154 (Hard)	Item accepted
4	0.743			0.181 (Hard)	Item accepted



The research procedure followed qualitative research protocols, which included the following steps: (1) ensuring all research-related legalities are addressed, (2) developing a non-story MPS test and listing opening questions for the interview, (3) validating the test instrument, (4) analyzing test items, (5) revising the test, (6) collecting data, (7) analyzing the data, (8) presenting the findings, and (9) drawing conclusions.

#### d. Data Collection and Analysis Techniques

In this research, the data collection techniques employed included tests and non-tests. The test technique has been previously explained. Non-test techniques involved observation, documentation, and unstructured interviews. The researchers also used the triangulation method to ensure the truth value of the data (Carter et al., 2014; Lopez & Whitehead, 2014).

The observations were conducted by observing students during a mathematics class. The documents collected comprised student answer sheets, a list of attendees, and

students' score lists. Observations and document collections were conducted to gather background information and assess the student's potential data value contribution for interviews. Additionally, unstructured, in-depth phenomenological interviews were conducted to gather information on the participants' experiences, feelings, and beliefs (Groenewald, 2004).

Phenomenological interviews offer a glimpse into universal essences and eidetic structures (Aguas, 2022). In this study, the interviews were conducted through the Zoom meeting application. The participants permitted the interviews to be recorded, and the conversations were transcribed verbatim (Aagaard, 2017).

The data analysis phase followed Aguas' (2022) analysis steps, which combine Moustaka's transcendental phenomenological data analysis and Van Manen's hermeneutic approach. The researchers used the NVIVO 12 (trial version) software for the data analysis. The data analysis procedure by Aguas (2022) is outlined in Table 5.

**Table 5. Aguas' (2022) Data Analysis Procedure**

No	Step	Activity
1	Initial Transcription and Coding	a. Conduct bracketing. b. Transcribe all words of the interviews.
2	Significant Statements	a. Use color coding. b. Maintain a smooth coding flow. c. List all significant statements that are not repetitive and do not overlap to develop initial categories. d. Discard statements that do not contribute to the understanding the participants' statements when developing categories and subcategories.
3	Initial Categories and Sections	a. Analyze data based on research questions. b. Use coding to reduce data into themes. c. Start data coding by performing initial coding, magnitude coding, subcoding, holistic coding, structural coding, process coding, descriptive coding, and in-vivo coding (read in Aguas (2022) for all of coding definitions).
4	Refining Categories From Selective Coding	a. Create broad and specific categories. b. Organize categories. c. Repeat previous activity.

No	Step	Activity
5	Initial and Final Themes of the Final Categories	<ul style="list-style-type: none"> <li>a. Incorporate all relevant categories into the phenomenon.</li> <li>b. Develop initial themes from categories in previous activities.</li> <li>c. Organize all categories into themes.</li> </ul>
6	Description of Texture and Structure, as well as Interpretation of Phenomena	<ul style="list-style-type: none"> <li>a. Produce texture and structure descriptions and interpretations for each theme.</li> <li>b. Include in-vivo examples to represent participants' experiences of the phenomenon under study.</li> </ul>
7	Combined Description and Interpretation of the Essence of Phenomena	<ul style="list-style-type: none"> <li>a. Perform a combined description of the target phenomenon and adjust the activities in the previous steps through iterative analysis.</li> </ul>
8	Validation of Participants' Inputs	<ul style="list-style-type: none"> <li>a. Send the analysis results to the participants.</li> <li>b. Ask the participants about the analysis results.</li> <li>c. Be aware that participants may or may not respond to the request.</li> </ul>

### 3. Results and Discussion

The final step of the data analysis process involved presenting the results to the four participants. The participants did not identify any discrepancies between the interview responses and the analysis findings. The authenticity of the answer sheets was confirmed through interviews with the participants. According to [Aguas \(2022\)](#), this step helps mitigate researcher bias. Meanwhile, the Rott-Specht-Knipping's MPS model helps with conducting an unbiased analysis of the student's answer sheets due to its descriptive and non-linear nature ([Rott et al., 2021](#)). Unlike normative MPS models, the Rott-Specht-Knipping model does not impose rigid steps, allowing for a more open analysis process.

The following section presents the results of the final analysis and discusses them.

#### a. Students' MPS Process

The descriptive MPS model by Rott-Specht-Knipping consists of an understanding (analysis) phase, an exploration phase, a planning phase, an implementing phase (sometimes combined with planning), and a verification phase. This model captures the full range of possible processes in students' MPS in a non-linear way, making it more flexible than the normative MPS model. Table 6 presents indicators for each MPS phase based on Rott-Specht-Knipping's framework.

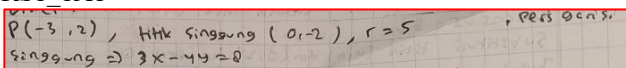
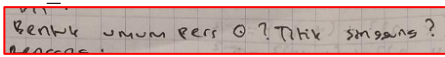

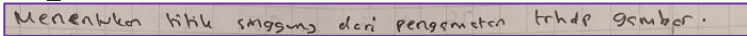
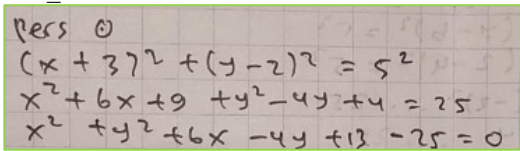
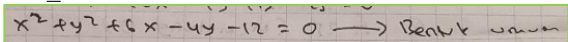
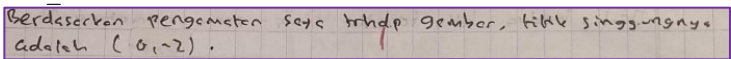
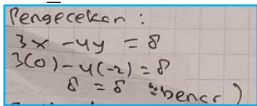
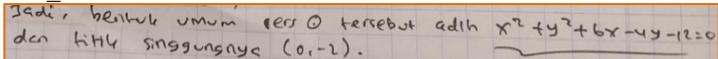
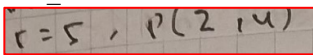
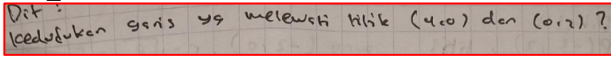
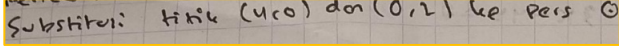
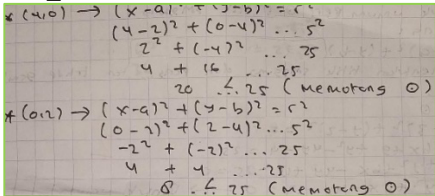
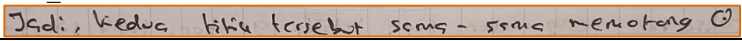
**Table 6. MPS Codes, Phases, and Indicators Based on Rott-Specht-Knipping's Model**

Code	Phase	Indicator
A	Analysis	<ul style="list-style-type: none"> <li>a. Presenting what is asked.</li> <li>b. Presenting important information.</li> </ul>
E	Exploration	<ul style="list-style-type: none"> <li>a. Presenting an example/case/particular method as a plan to answer the question and written in an unstructured manner.</li> <li>b. Presenting something that is not an example as a plan to answer the question and is written in an unstructured manner.</li> </ul>
P	Planning	<ul style="list-style-type: none"> <li>a. Presenting plans in a structured manner without involving examples and/or non-examples.</li> <li>b. Presenting plans by adapting/combining previously known procedures.</li> </ul>
I	Implementation	<ul style="list-style-type: none"> <li>a. Presenting procedures based on the exploration or/and planning phases.</li> <li>b. Presenting the final solution based on the procedures conducted.</li> </ul>
V	Verification	<ul style="list-style-type: none"> <li>a. Presenting another method to check solutions.</li> </ul>

Next, after identifying the indicators for each MPS phase, the students' answer sheets were analyzed based on these indicators to determine the MPS process followed.

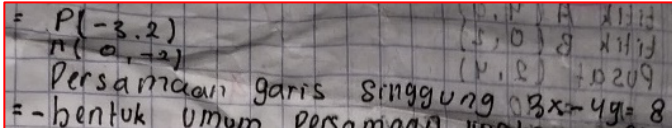
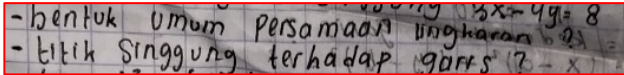
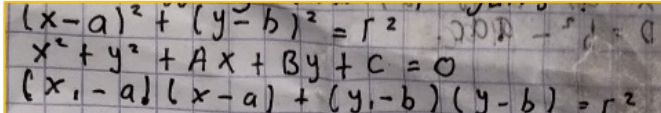
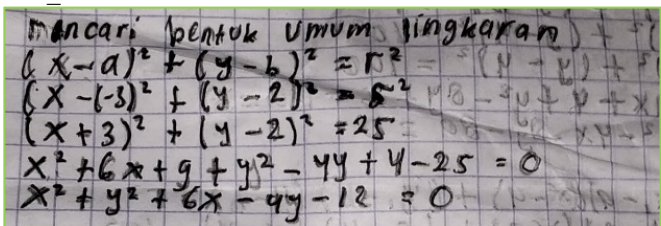
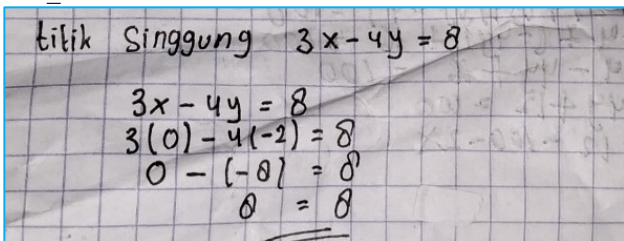
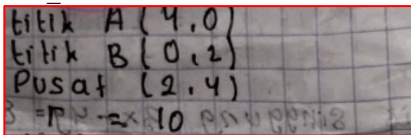
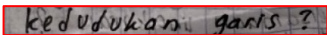
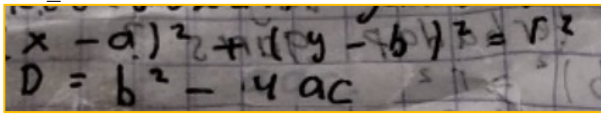
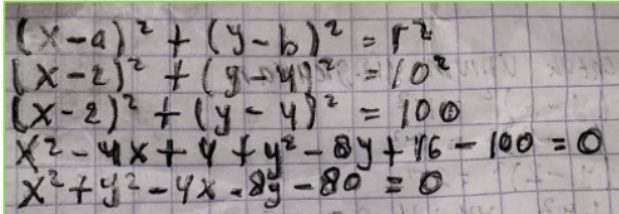
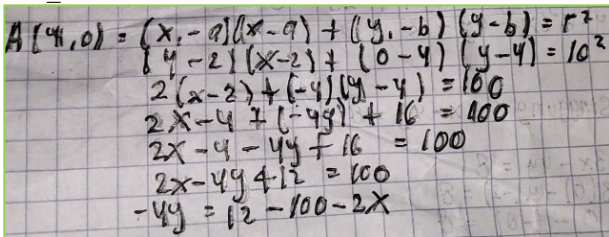
Table 7 presents the sequence of participants' MPS process in answering mathematics problems as reflected in their answer sheets.

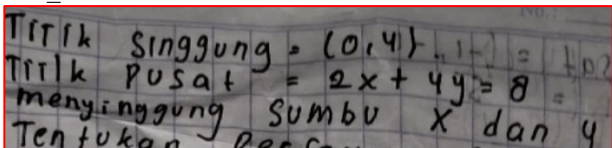
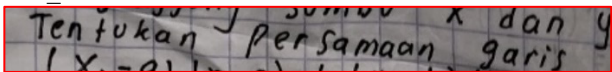
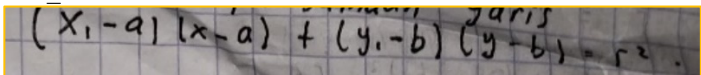
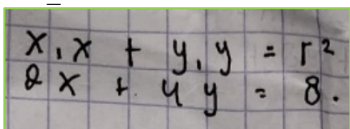
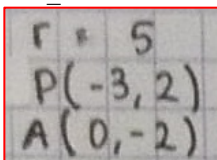
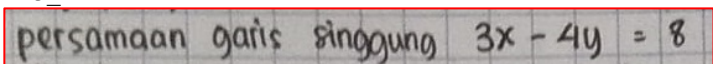
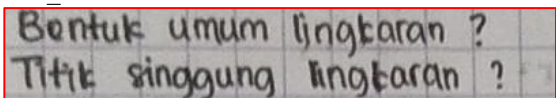
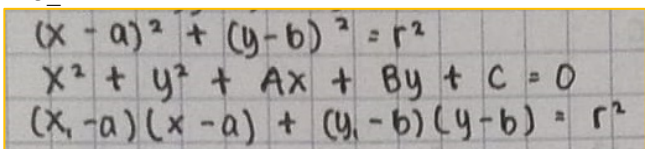
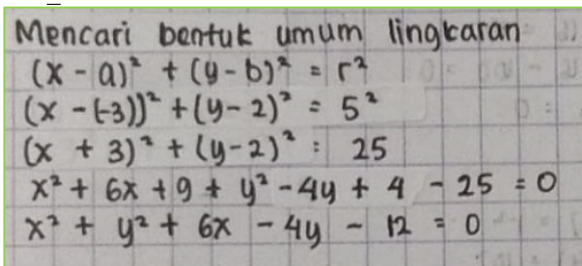
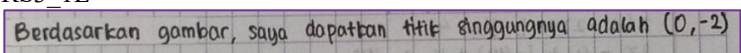
Table 7. Students' MPS Process

Participant	Process	Coded Answer Sheet
RS1	General:	Item 1:
	$A \rightarrow P \rightarrow E \rightarrow I$	RS1_1A1
	$\rightarrow E \rightarrow V$	
	Question 1:	RS1_1A2
	$A \rightarrow P \rightarrow I$	
	Question 2:	RS1_1P
	$A \rightarrow E \rightarrow V$	
		RS1_1E1
		
		RS1_1I1
		
		RS1_1I2
		
		RS1_1E2
		
		RS1_1V
		
		RS1_1Affirmation
		
	$A \rightarrow P \rightarrow I$	Item 2:
		RS1_2A1
		
		RS1_2A2
		
		RS1_2P
		
		RS1_2I
		
		RS1_2Affirmation
		
	$A \rightarrow P \rightarrow I \rightarrow V$	Item 3:
		RS1_3A1



Participant	Process	Coded Answer Sheet
RS1		<p>Diket:</p> <p>Titik Singgung <math>(0,4)</math>, Pers <math>\odot = 2x + 4y = 8</math>, <math>r = \sqrt{8} = 2\sqrt{2}</math></p> <p>RS1_3A2</p> <p>Persamaan garis singgung <math>\odot</math> ?</p> <p>RS1_3P</p> <p><math>(m,n) \rightarrow (x-a)^2 + (y-b)^2 = r^2</math>  <math>\rightarrow (m-a)(x-a) + (n-b)(y-b) = r^2</math></p> <p>RS1_3I1</p> <p> <math>0,4 \rightarrow (x-2)^2 + (y-4)^2 = 8</math>  <math>\rightarrow (0-2)(x-2) + (4-4)(y-4) = 8</math>  <math>-2(x-2) + 0(y-4) = 8</math>  <math>-2x-2 = 8</math>  <math>-2x = 10</math>  <math>x = -5</math> </p> <p>RS1_3I2</p> <p> <math>2x + 4y = 8</math>  <math>2(-9) + 4y = 8</math>  <math>-18 + 4y = 8</math>  <math>4y = 8 + 18</math>  <math>4y = 26</math>  <math>y = \frac{26}{4}</math>  <math>y = \frac{13}{2}</math> </p> <p>RS1_3V</p> <p> <math>2x + 4y = 8</math>  <math>2(-9) + 4\left(\frac{13}{2}\right) = 8</math>  <math>-18 + 26 = 8</math>  <math>8 = 8</math> (benar) </p> <p>RS1_3Affirmation</p> <p>Jadi, Persamaan garis singgung <math>\odot</math> tersebut adalah <math>x = -9</math> dan <math>y = \frac{13}{2}</math></p>
	$A \rightarrow P \rightarrow I \rightarrow V$	<p>Item 4:</p> <p>RS1_4A1</p> <p><math>P(-1,-1)</math>, <math>r = 5</math>, <math>y = -x - 1</math>, Titik Singgung <math>(4,-1)</math></p> <p>RS1_4A2</p> <p>Persamaan garis singgung <math>\odot</math> !</p> <p>RS1_4P</p> <p>Mencari Pers <math>\odot</math>, lalu</p> <p>RS1_4I1</p> <p>           Pers <math>\odot</math>  <math>(x-a)^2 + (y-b)^2 = r^2</math>  <math>(x+1)^2 + (y+1)^2 = 5^2</math>  <math>(x+1)^2 + (y+1)^2 = 25</math> </p> <p>RS1_4P2</p> <p> <math>(4,-1) \rightarrow (x+1)^2 + (y+1)^2 = 25</math>  <math>\rightarrow (4+1)^2 + (-1+1)^2 = 25</math>  <math>5(x+1) = 25</math>  <math>5x+5 = 25</math>  <math>5x = 20</math>  <math>x = 4</math> </p> <p>RS1_4V</p> <p> <math>x+1 = 5</math>  <math>4+1 = 5</math>  <math>5 = 5</math> (benar) </p> <p>RS1_4Affirmation</p> <p>Jadi, Pers garis singgung <math>\odot</math> tersebut adalah <math>x+1 = 5</math>  <math>x = 4</math></p>
RS2	General: $A \rightarrow P \rightarrow I \rightarrow V$	<p>Item 1:</p> <p>RS2_1A1</p>

Participant	Process	Coded Answer Sheet
	Question 1: $A \rightarrow P \rightarrow I$	
	Question 2: $A \rightarrow V$	RS2_1A2 
		RS2_1P 
		RS2_1I 
		RS2_1V 
	$A \rightarrow P \rightarrow I$	Item 2: RS2_2A1 
		RS2_2A2 
		RS2_2P 
		RS2_2I1 
		RS2_2I2 
	$A \rightarrow P \rightarrow I$	Item 3:

Participant	Process	Coded Answer Sheet
RS2		RS2_3A1 
		RS2_3A2 
		RS2_3P 
		RS2_3I 
	N/A	Item 4: Unfinished.
RS3	General: $A \rightarrow P \rightarrow I \rightarrow E$ $\rightarrow V$	Item 1: RS3_1A1 
	Question 1: $A \rightarrow P \rightarrow I$	RS3_1A2 
	Question 2: $A \rightarrow E \rightarrow V$	RS3_1A3 
		RS3_1P 
		RS3_1I 
		RS3_1E 
		RS3_1V

## Participant Process Coded Answer Sheet

$$3x - 4y = 8$$

$$3(0) - 4(-2) = 8$$

$$8 = 8$$

 $A \rightarrow P \rightarrow I$ 

Item 2:

RS3\_2A1

RS3\_2A2

titik A(4,0)  
titik B(0,2)  
pusat P(2,4)  
 $r = 10$

Kodudikan garis?

RS3\_2P

$$(x-a)^2 + (y-b)^2 = r^2$$

$$D = b^2 - 4ac$$

RS3\_2I1

RS3\_2I2

$$(x-a)^2 + (y-b)^2 = r^2$$

$$(x-2)^2 + (y-4)^2 = 10^2$$

$$x^2 - 4x + 4 + y^2 - 8y + 16 = 100$$

$$x^2 - 4x + 4 + y^2 - 8y + 16 - 100 = 0$$

$$x^2 + y^2 - 4x - 8y - 80 = 0$$

titik A(4,0)  
 $(x_1-a)(x-a) + (y_1-b)(y-b) = r^2$   
 $(4-2)(x-2) + (0-4)(y-4) = 10^2$   
 $2(x-2) + (-4)(y-4) = 100$   
 $2x - 4 + (-4y) + 16 = 100$   
 $2x - 4 - 4y + 16 = 100$   
 $2x - 4y + 12 = 100$

$$-4y = 100 - 12 - 2x$$

$$-4y = -2x + 88 \quad : -4$$

$$y = \frac{1}{2}x + 22$$

RS3\_2I3

$$x^2 + y^2 - 4x - 8y - 80 = 0$$

$$x^2 + \left(\frac{1}{2}x + 22\right)^2 - 4x - 8\left(\frac{1}{2}x + 22\right) - 80 = 0$$

$$x^2 + \frac{1}{4}x^2 + 22x + 484 - 4x - 4x - 176 - 80 = 0$$

RS3\_2I4

RS3\_2I5

$$\frac{5}{4}x^2 + 14x + 228 = 0$$

$$D = b^2 - 4ac$$

$$= 14^2 - 4\left(\frac{5}{4}\right)(228)$$

$$= 196 - 1140$$

$$= -944 \Rightarrow$$

RS3\_2I6

$$D = -944$$

$$-944 < 0$$

tidak mempunyai  
akar nyata

 $A \rightarrow P \rightarrow I$ 

Item 3:

RS3\_3A1



Participant	Process	Coded Answer Sheet
		<p>titik singgung (0,4)</p> <p>titik pusat <math>2x + 4y = 8</math></p> <p>menentukan sumbu x &amp; y</p> <p>RS3_3A2</p> <p>Persamaan garis singgung</p> <p>RS3_3P</p> <p><math>(x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2</math></p> <p>RS3_3I</p> <p><math>x_1x + y_1y = r^2</math></p> <p><math>2x + 4y = 8</math></p>
	$A \rightarrow P$	<p>Item 4:</p> <p>RS3_4A1</p> <p>titik pusat <math>P(-1,-1)</math></p> <p><math>r = 5</math></p> <p>garis singgung <math>y = -1x - 1</math></p> <p>RS3_4A2</p> <p>persamaan garis singgung</p> <p>RS3_4P</p> <p><math>(x_1 - a)(x - a) + (y_1 - b)(y - b) = r^2</math></p> <p>Unfinished.</p>
RS4	<p>General:</p> <p><math>A \rightarrow P \rightarrow V \rightarrow I</math></p> <p>Question 1:</p> <p><math>A \rightarrow P \rightarrow I</math></p> <p>Question 2:</p> <p><math>A \rightarrow V</math></p>	<p>Item 1:</p> <p>RS4_1A1</p> <p>6 mar/24</p> <p>titik pusat = <math>(-3, 2)</math></p> <p>titik singgung = <math>(0, -2)</math></p> <p>pers. garis singgung = <math>3x - 4y = 8</math></p> <p>RS4_1A2</p> <p>bentuk umum persamaan lingkaran dan titik singgungnya terhadap garis!</p> <p>RS4_1P</p> <p>mencari jari<sup>2</sup>,</p> <p>masukkan ke dalam rumus,</p> <p>cari bentuk umum 2 titik singgung</p> <p>RS4_1V</p> <p>titik garis singgung</p> <p><math>3x - 4y = 8</math></p> <p><math>3(0) - 4(-2) = 8</math></p> <p><math>0 - (-8) = 8</math></p> <p><math>8 = 8</math></p> <p>RS4_1I</p> <p><math>(x+3)^2 + (y-2)^2 =</math></p> <p><math>x^2 + 6x + 9 + y^2 - 4y + 4 = 25</math></p> <p><math>x^2 + y^2 + 6x - 4y + 13 = 25</math></p> <p><math>x^2 + y^2 + 6x - 4y - 12 = 0</math></p>
	$A$	<p>Item 2:</p>



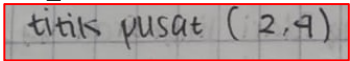
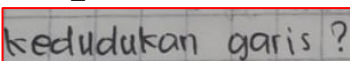
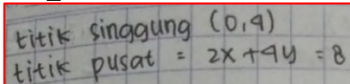
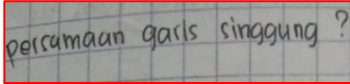
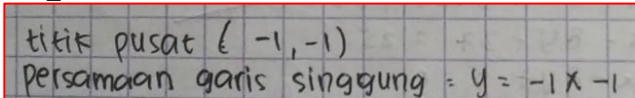
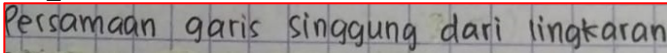
Participant	Process	Coded Answer Sheet	
		RS4_2A1	RS4_2A2
			
		Unfinished.	
	A	Item 3: RS4_3A1	
			
		RS4_3A2	
			
		Unfinished.	
	A	Item 4: RS4_4A1	
			
		RS4_4A2	
			
		Unfinished.	

Table 7 shows that only RS1 provided complete answers to all the problems. RS1 also utilized validation methods to confirm her answers, except for item 2 (problem 2). Meanwhile, RS2 and RS3 did not find a solution for problem 4, while RS4 only succeeded in solving problem 1.

Moreover, RS1 demonstrates greater consistency in executing the MPS process than RS2, RS3, and RS4 by following the  $A \rightarrow P \rightarrow I \rightarrow V$  process, except for problem 2. RS1 and RS3 also engaged in the exploration phase at problem 1, while RS2 and RS4 did not. Additionally, RS2 and RS3 are more prominent in the MPS  $A \rightarrow P \rightarrow I$  process, whereas RS4 excels in the analysis phase.

Furthermore, RS1 exhibits the most transitions between phases, namely  $A \rightarrow P \rightarrow E \rightarrow I \rightarrow E \rightarrow V$  for addressing problems 1 and 2. These processes align with the flow and transitions outlined in Rott-Specht-Knipping's descriptive MPS model (Rott et al., 2021). Students need to know that solving a math problem involves a flexible and adaptive approach, where they can navigate back

and forth to determine the most effective solution (García et al., 2019).

#### b. The Students' Errors

The online interviews were conducted at the end of June 2024. Once all participants joined the Zoom call, they were placed in breakout rooms for individual interviews. The participants were enthusiastic when asked about their well-being, demonstrating their willingness and readiness for the interviews. On average, the interviews lasted approximately 22 minutes.

All participants were highly active students in the classroom and had good communication skills. They also remembered the topics tested in the Mathematics Specialization subject, including circle formulas, positions of lines, and the tangents to a circle equation. They found the instructions on the question sheet, particularly in the recommended answer format section, to help answer non-story mathematics problems. RS3 provided the following statement:

"It was useful, sir. For example, if we don't have enough information, we won't know what to ask about, and what plans to make. So, we also can't answer the question. Of course, the first thing we need to do is understand the provided information first, then we can figure out what the question is asking about. After we know what is being asked, we can determine which formula we can use."

However, as shown in Table 7, it is evident that not everyone adheres to the recommended answer format, including RS3. This finding suggests that the students faced obstacles that prevented them from presenting answers in the recommended format.

Nevertheless, all participants clearly understood the questions posed in each problem. RS1, RS2, and RS3 confidently stated they had no issues and answered problem 1 correctly. RS4 also claimed the same but struggled with problem 1.

RS1, RS2, and RS4 encountered difficulties in problems 2, 3, and 4, while RS3 faced challenges in points 3 and 4. These initial findings shed light on potential obstacles, prompting a deeper investigation. The results of the analysis of students' errors in solving mathematics problems are presented in Table 8.

Table 8. The Students' Error

Participant	Item/Problem	Error Type	Coding Source
RS1	2	a. Analysis Error	a. RS1_2A1, "[Point on graph] for possible checking"
		b. Planning Error Due to Carelessness	b. RS1_2P, "Is that the position of the point, sir? The question is about the position of the line."
		c. Implementation Error	c. RS1_2I, "Find the equation first to get to the general form. Then, just enter the point (4,0) one by one until you get the Y. Uh, how's it going?"
	3 & 4	a. Analysis Error	a. RS1_3A1, RS1_4A1, "How do I do this, sir? I don't know, sir."
		b. Planning Error	b. RS1_3P, RS1_4P, "Y minus Y1 is the same as MX minus X1."
		c. Implementation Error	c. RS1_3I1, RS1_3I2, RS1_4I1, RS1_4I2
		d. Verification Error	d. RS1_3V, RS1_4V
RS2	2	a. Analysis Error	a. "Yes, which point do you want to use? That's how you do it."
		b. Planning Error	b. RS2_2P
		c. Implementation Error	c. RS2_2I1, RS2_2I2
	3	a. Analysis Error	a. RS2_3A1, "I don't understand, sir."
		b. Implementation Error	b. RS2_3I
	4	a. Analysis Error	a. "Yes, the equation of a line. Same tangent point (4, -1)."
		b. Planning Error	b. $(x + 1)^2 + (y + 1)^2 = 2r^2$ , is that correct, sir?"
RS3	2	a. Analysis Error	a. "[The dots] are to find the position of the line."
		b. Implementation Error	b. RS3_2I1, RS3_2I2, RS3_2I3
		c. Verification Error	c. "[Another way to answer] I didn't think of that, sir."
	3	a. Analysis Error	a. RS3_3A1, "Yes [does not understand required information]."
		b. Planning Error	b. RS3_3P
		c. Implementation Error	c. RS3_3I
	4	a. Analysis Error	a. RS3_4A1
		b. Verification Error	b. "I don't know [how to check it], sir."

Participant	Item/Problem	Error Type	Coding Source
RS4	1	a. Analysis Error	a. "I don't know if it's in the picture [the graph, the radius], sir."
		b. Implementation Error	b. RS4_1I
		c. Verification Error	c. "I don't know [how to determine the tangency point], sir."
	2, 3, & 4	Analysis Error	RS4_3A1, RS4_4A1, "Yes, [I don't understand the information in the question]", "I don't know [the purpose of the line that cuts the circle], sir."

Table 8 shows that the initial error occurred because the participant made an analysis error. Lester's research indicates that an individual's analysis or understanding phase significantly impacts their MPS performance (Pugalee, 2001). The analysis phase, which involves understanding and defining the problem, is crucial in the MPS process and engages various parts of the brain (Anderson et al., 2014). Therefore, if this phase is flawed, students may encounter errors when progressing to the next phase. The relationship between all types of errors is shown in Figure 1.

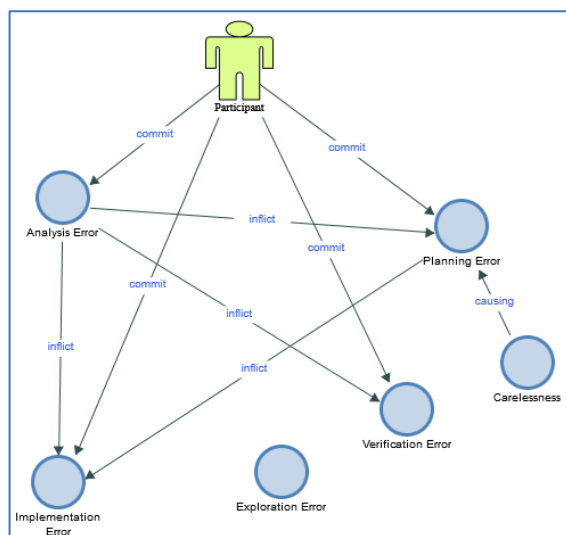


Figure 1. Relationship Between Each Type of Error

Figure 1 shows that no exploration errors were detected because the exploration phase was exclusively conducted on problem 1 by RS1 and RS3 (Table 7). Additionally, there were planning errors stemming from carelessness. This finding aligns with the definition of

carelessness provided by Clements (1982), where RS1 responded accurately during the planning phase in the interview but presented the work incorrectly on the answer sheet.

White (2005) also categorizes carelessness as a common error in solving mathematics problems. In addition, (Ken) Clements (1980) discovered that carelessness can lead to other errors. A hierarchy of participant error types was established for answering non-story mathematics problems, as shown in Figure 2.

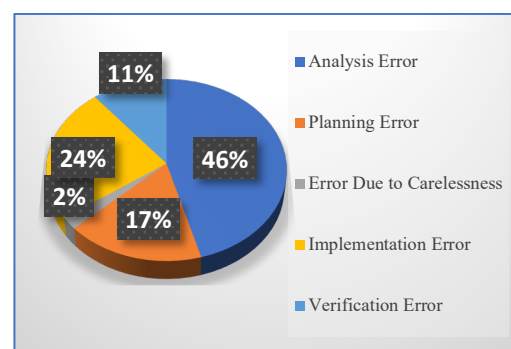


Figure 2. Hierarchy of Error Types

In Figure 2, the percentage of verification errors was the lowest, as some participants did not complete the verification phase for certain problems. Some students dislike using verification methods to check their answers (García et al., 2019). This sentiment has been supported by previous studies, such as those conducted by Cleary & Chen (2009) and García et al. (2016).

### c. Factors Contributing to Students Errors

After identifying the types of errors in responding to non-story mathematics problems, the participants were interviewed about the factors that led to these errors. Before the interview, RS1 acknowledged some of her errors by promptly stating, "That was incorrect [in point 2, implementation error]." The other three participants did not exhibit the same behavior. Consequently, the participants were categorized into two themes: those conscious of their errors and those who were not. This finding was also found in previous studies, such as the study by [Adinda et al. \(2021\)](#).

The students' ability to realize errors can be trained by giving them self-reflective activities. Engaging students in self-reflective activities can enhance their ability to recognize errors, improving focus, motivation, and learning engagement ([Karaali, 2015](#)). This activity can also improve the students' MPS performance ([Kwon & Jonassen, 2011](#)).

Next, the interview transcripts were analyzed using the Aguas steps, and all previous types of coding were incorporated. Following the analysis, the relationship between the factors was derived from the identified themes, categories, and sub-categories. The results of this analysis are presented in Figure 3.

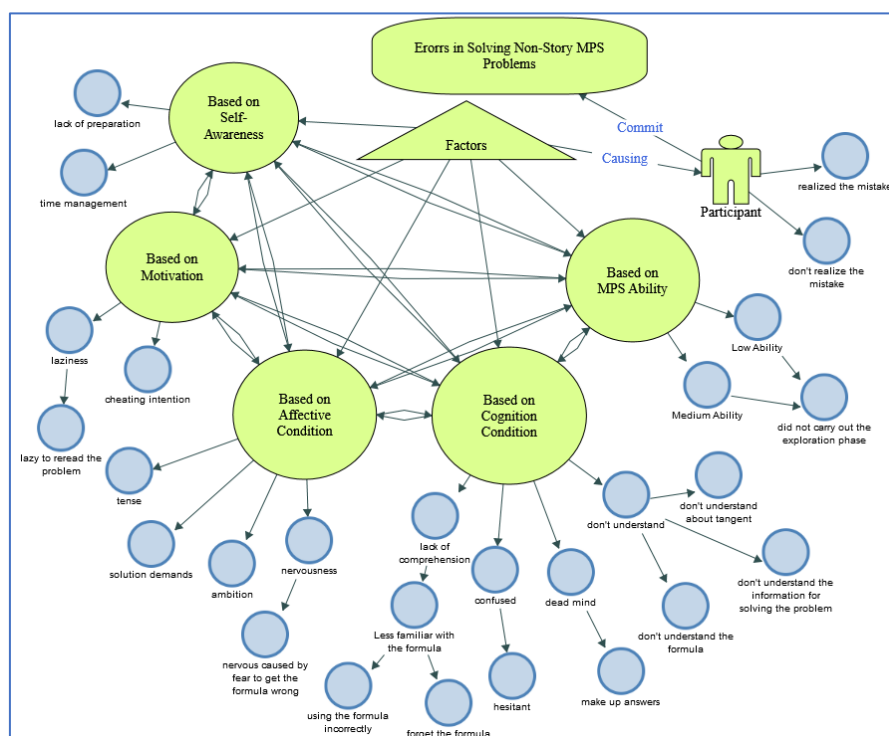


Figure 3. Relationship Between Factors in Solving Non-Story Mathematics Problems

Figure 3 shows five themes affecting the students' errors in solving non-story MPS problems: MPS abilities, cognitive conditions, affective conditions, motivation, and self-awareness. Research has shown a reciprocal relationship between cognitive and affective conditions ([Trezise & Reeve, 2014](#)). This finding is supported by case studies

conducted by [Furinghetti and Morselli \(2009\)](#), which validate Schoenfeld's assertion that cognition and affect in MPS are interconnected. In addition to these two themes, other themes may also be interrelated.

### 1) MPS Ability (MPSA) Domain

For the error factor based on MPS ability (MPSA), almost all participants did not explore the mathematical problems (Table 7). The exploration phase in solving mathematics problems typically involves visual representations of crucial information. Skipping this phase can reflect students' MPS abilities, as stated by Krawec (2014).

The lack of exploration is considered an error factor because, according to Schoenfeld, exploration is the heuristic core of MPS (Rott et al., 2021). Rott et al. (2021) also indicated that the exploration phase was observed at least once in non-routine problems. These studies highlight the significance of the exploration phase in addressing mathematics problems.

Furthermore, it was observed that in several instances, participants completed only the planning phase of the problem-solving process without progressing to the exploration phase, which is crucial for thoroughly analyzing and testing potential solutions. This incomplete approach significantly impacted their ability to arrive at correct solutions for the non-story mathematics problems. Specifically, all problems attempted without the exploration phase resulted in incorrect or incomplete solutions, highlighting the importance of engaging with the exploration phase to verify and refine initial ideas (Hardini et al., 2021; Darmawan & Suparman, 2019). Moreover, a pattern of errors was evident in those problems that had not undergone the exploration phase, suggesting that skipping this step led to fundamental misunderstandings or overlooked aspects of the problems (Khusnani et al., 2023; Kurniawan et al., 2022). The only exception to this trend was observed in RS2 in Problem 1 (refer to Table 8), where the participant's planning alone appeared sufficient to generate the correct solution. These findings underscore the critical role of the

exploration phase in the problem-solving process, as it allows for the testing and modification of initial plans, ensuring a more robust and accurate solution.

### 2) Cognition Domain

For the cognition domain, the following factors contributed to the students' errors: lack of understanding, confusion, dead mind, and misinterpretation. The issue of confusion was also identified in a study conducted by Ruzlan, Rosalinda, and Arsaythamby in 2013 (Veloo et al., 2015). These factors can hinder comprehension of concepts or schema. Substantial evidence indicates that domain-specific knowledge, in the form of schemas or concepts, is the key factor that sets experts apart from novices in MPSA (Sweller, 1988; Veloo et al., 2015).

Furthermore, evidence was found of the influence of other cognitions in terms of students' cognitive styles (field-dependent, intermediate, and independent) influencing MPSA linearly (Ulya, 2015). If there are issues with understanding the concepts or schemas, students may struggle to grasp the problems and may even be unable to attempt to solve them (Rhodes, 1987). For instance, RS4 faced challenges in comprehending concepts related to points 2, 3, and 4, limiting progress to the analysis phase (refer to Table 7 and Table 8).

### 3) Affective Domain

The identified factors related to affective conditions include feeling tense, the demand to answer, ambition, and nervousness. This domain has been extensively studied in connection with MPS, as demonstrated in research by DeBellis and Goldin (2006), which categorizes affection into beliefs, attitudes, emotional states, and ethics/morals/norms. Based on the interview results, only RS3 did



not exhibit any affective conditions, while RS1, RS2, and RS4 were nervous.

RS1 felt nervous because she felt she was demanded to answer the question, she said; "I was just nervous because it was like I was being forced [pressured]. It seems like the question is difficult," and "The important thing is to answer." RS2 felt tense, nervous, and ambitious, she said; "Nervous and afraid of making errors. I used the wrong formula, sir," and "I want to try to get good grades, sir." Meanwhile, RS3 did not feel any affection, and RS4 felt nervous. Apart from RS3, the other participants expressed epistemic emotional states, namely emotions whose object is focused on knowledge and meaning (Muis et al., 2015).

#### 4) Motivation Domain

Motivation plays a significant role in students' MPSA and performance. Irhamna et al. (2020) found that motivation accounted for 15.8% of students' MPSA. Similarly, Fatimah et al. (2019) reported that motivation significantly impacted students' MPSA. Research has shown that motivation, mediated by self-efficacy, impacts students' MPS performance (Özcan & Eren Gümüş, 2019).

In contrast to the research by (Ken) Clements (1980), which considered motivation as a type of error, this study identified motivation as a contributing factor to errors, and is divided into two parts: laziness and cheating intention. Laziness was found in RS1 as she said she was "Lazy to re-read [questions]" in point 2. Thus, the student was careless, which led to planning errors.

Moreover, Pavlin-Bernardić et al. (2017) found that motivation is closely linked to active cheating. Cheating intention is key in actively seeking to cheat to enhance one's success. Cheating intentions were found in RS2, RS3, and RS4, who admitted they had such intentions. The three participants stated

that they failed to act on their intentions. RS2 said this was due to the lack of time; she said:

"Maybe, yesterday it was number four, because it wasn't filled, right. [Whispering to friend] Help! But [in the end] it wasn't filled either. Because the time is up".

RS3 and RS4 stated that they failed to act on their cheating intention because their friends also did not find a solution, as RS3 said

"There are a few codes [or whispering to friends]. In the end, he/she didn't know either, so we just had to answer it ourselves".

#### 5) Self-Awareness Domain

For the self-awareness domain, two factors were identified: time management and lack of preparation. According to Blakemore and Frith (2003), self-awareness of certain actions is influenced by various aspects, such as awareness of intention and a sense of agency. Self-awareness accuracy is also expected to improve with age (Demetriou & Kazi, 2006). Therefore, time management and lack of preparation fall under these two overlooked aspects and that there is an element of intentionality carried out by students.

Other than RS4, the other three participants complained that they ran out of time to solve the problems. RS2 mentioned, "Yeah, there was not enough time, sir. Because I was thinking about the previous question, sir. So, I didn't get to fill in number 4." Additionally, RS1 and RS4 felt that they had not prepared themselves enough to face these non-story mathematics problems. RS4 stated, "Yes, I didn't prepare myself well for this test, sir."

#### 4. Conclusion

This study aimed to understand student's errors in solving non-story mathematics problems using the phenomenology framework. The errors the high school students made

while solving these problems were analyzed using the mathematics problem solving (MPS) process outlined by Rott-Specht-Knipping, and the factors contributing to these errors were identified. The study identified various types of errors, such as analysis errors, planning errors, carelessness in planning, implementation errors, and verification errors. Analysis errors were the most common, while errors due to carelessness were the rarest. The MPS process with the most transitions is Analysis → Planning → Exploration → Implementation → Exploration → Verification. Moreover, this study found that conducting an exploration phase can lead to the correct solution. The analysis showed that errors frequently arise when students neglect the exploration phase of the MPS process, impacting their performance. In other words, errors were predominantly found in problems solved without engaging in the exploration phase. The factors contributing to all these errors fall into five domains: MPS ability (MPSA), cognition, affection, motivation, and self-awareness. These domains all play crucial roles in students' errors and the MPS process, with cognition and affection being closely linked. Other domains may also be interrelated. This study serves as a guide for conducting an error analysis using a descriptive MPS model that other researchers can replicate. It offers valuable insights for teachers, particularly mathematics educators, emphasizing the significance of error analysis. It can also inform educational policies regarding mathematics teaching by recommending the consistent integration of error analysis after completing a mathematics topic for appropriate remediation. The remediation program could also include self-reflective activities to enhance students' MPSA and error recognition abilities. However, this study is limited to geometry topics, specifically circle analysis subtopics and non-story mathematics problems.

Recommendations for future research include exploring word problems in different topics and incorporating technology into students' MPS processes. Further investigation into the exploration phase and its impact on students' MPS and error rates is also suggested. Future research should also aim to validate assumptions regarding the interplay of the five domains of error factors identified in this study.

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