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Integrated Production and Distribution Planning using Mathematical Model for Maximizing Profit

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Abstract. *In three-dimensional concurrent engineering context, product, process, and supply chain designs are approached by the imperative of concurrency principle. The imperative of concurrency proposes product architecture and supply chain architecture conducted simultaneously. The detailed product design and production process design are conducted simultaneously. While manufacturing system design is conducted simultaneously with logistic system design. This study concerns on integrating production planning with distribution planning in a new plastic bags plant. Since the plant is relatively new, the number of orders is still scarce. Therefore, production planning becomes critical. Producing more products than required creates inventories while producing as many as the order required creates an intermittent production schedule. A mathematical is developed to simultaneously schedule production and distribution to maximize profit. The model provides solutions on the number of materials to be delivered, the number of products produced and delivered within its period for the planning horizon.*

Keywords: *integrated production and distribution planning; 3DCE; mathematical model; integrated decision making*

I. Introduction

A new plastic-producing company produces plastic bags. The demands are still volatile and below their expected production capacity. To meet its demands this company requires several materials supplied by its suppliers.

The company has to prepare a production schedule that meets the demands. More production will result in over-inventory, while less production will decrease demand fulfillment. Furthermore, the company must ensure the number of materials required from the suppliers that may cause over inventory in material storage or shortage in another way.

Moreover, the issue comes also at the downstream of the supply chain whether the company should produce as the demand required that will increase the distribution cost or produces for several demands that will increase the final product inventory. Therefore, integration is needed to reduce the streamline costs and deal with the product demand patterns including production quantities and deliveries (Herlina, et al., 2022).

Another urgency for integrating production and distribution planning is the problem often comes late when the distribution is not planned simultaneously with the production. For example, the produced product might be delivered in the wrong quantities at the wrong time when it plan is not conducted simultaneously (Fahimnia, et al., 2013). An optimal production schedule without synchronization with the distribution plan often ends in inventory problems or distribution costs or customer satisfaction. Thus, the integration of production scheduling and distribution planning becomes necessary. This shows that production and distribution planning is the main process in the supply chain (Lee & Sook, 2000) and causes the largest costs in the process (Rafiei, et al., 2018).

This study basically provides a mathematical model that integrates production and distribution planning in a plastic bag mini-industry consisting of three echaelons: suppliers, a single manufacturer, and distributors. The integration considers the Three-Dimensional Concurrent Engineering perspective that suggests the Imperative of Concurrency (Fine, et al., 2005) principle which is the Focus Principle. The focus

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principle suggests the manufacturing system (where the production schedule is planned) is designed simultaneously with the logistics & coordination systems (where the distribution is planned) (Ilhami, et al., 2020).

The proposed model is a quantitative model for simultaneously production and distribution planning to maximize its profit. Therefore, based on the imperative of concurrency, this model is included in the Focus domain which deals with manufacturing systems and logistics & coordination systems.

The previous models underlying the proposed model are the integrated model to optimize the service level and the supply chain total cost (Rafiei, et al., 2018), to minimize inventory and transportation cost (Molefe, 2017), and to minimize time to market (Jafarian & Bashiri, 2014).

Furthermore, the integrated production planning and distribution deals with many industries (Guarnaschelli, et al., 2020) for example (1) the dairy industry (Liu, et al., 2019); (2) iron ore (Gharaei & Jolai, 2021); (3) automotive industry (Jabbarzadeh, et al., 2019); (4) pharmaceutical (He, et al., 2022); (5) spare part (Aazami & Saidi-Mehrabad, 2021); and (6) packaged vegetable.

The main contribution of the proposed model is it is the first integrated production and distribution planning model for manufacturer production under its production capacity.

II. Research Method

We develop the proposed model using the model development cycle (Anhalt & Cortez, 2015) that requires five steps: (1) creating a models map that provides the relevant models as the basic model or reference model; (2) model formulation where we determine all the required variables, parameters, constraints, and the objective functions; (3) model verification that ensure no error or mistake in the model; (4) model validation using sensitivity analysis to ensure the model behaves as predicted or logic; and (5) solution creation presents solution and results and its interpretation.

The purpose of the model is to deal with decisions in production and distribution integration planning such as the number of products produced including when, the number of products delivered to customers (Micro Small and Medium Enterprises / MSMEs), in what period, the number of materials, and when the materials delivered to the manufacturer from the suppliers.

Furthermore, the numerical example is presented to demonstrate the model's ability in dealing with the real problem. For solving the problem, the Lingo 19.0 software is used to provide the optimal solution for the real problem.

Finally, we perform the sensitivity analysis for validating the model by modifying the critical parameters and then analyzing the results.

III. Results and Discussion

Mathematical Model

As we mentioned earlier, the model solves decisions on the production and distribution planning of the three-echelon supply chain network. An integrated model considers three echelons consisting of multi-suppliers, a manufacturer, and multi-customers. The customers are micro, small & medium enterprises (MSME), and are divided into three regional groups. Figure 1 shows the supply chain network design.

The integration of production and distribution planning is formulated in Linear Programming (LP) to maximize its profit. The assumptions of the proposed model are as follows:

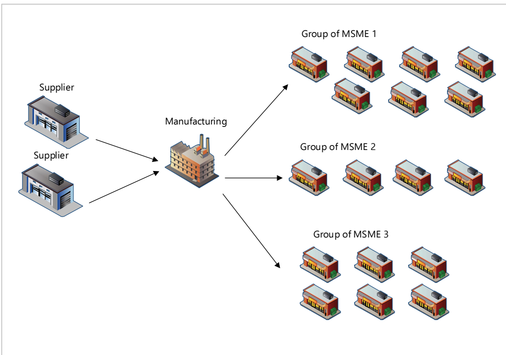

Figure 1. Network design

1. The production and delivery lead times are zero.
2. The planning period is discrete.
3. The Parameters are estimated and deterministic.
4. There is no inventory before the planning horizon.

The notations for the mathematical model are listed as follows:

Indices

i plastic products, $i \in I$
j raw materials, $j \in J$
r MSME, r $\in$ R
k supplier, k $\in$ K
t time period, $t \in T$

Parameters

P_{rj} price on MSME r for plastic product i
P_{kj} price from supplier k for raw material j
C_j the capacity of transporting raw materials j
C_i the capacity of transporting plastic product i
TC_{kj} transportation cost from supplier k for raw material j
B_i batch size of product i
TC_{rj} transportation cost to MSME r for product i
PC_i production cost of product i
D_{rit} demand of MSME r for product i in time period t
CP_{kj} delivery capacity from supplier k for raw material j
W_{ij} proportion of production of product i raw material j
Cap production capacity
I_{jt} inventory of raw material j in time period t
I_{it} inventory of product i in time period t
IC_j inventory cost of raw matrial j
IC_i inventory cost of product i

Decision variabels

Q_{it} quantity of product i that produce in time period t
S_{rit} quantity of production for MSME r for product i delivered in time period t
S_{kjt} the decision of supplier k to send a number of raw material j in time period t

Objective function

Z = maximize profit
Z = revenue – (raw material procurement cost + production cost + transportation cost + inventory cost)

- Revenue is presented as follows (Eq (1)):

$$= \sum_r \sum_i \sum_t S_{rit} P_{ri} \quad \textbf{(1)}$$

The revenue is described as the selling price of plastic bag products produced multiplied by the number of plastic bag products delivered to MSMEs.

- Raw material procurement cost is calculated as follows:

$$= \sum_k \sum_j \sum_t S_{kjt} P_{kj} + \sum_k \sum_j \sum_t \frac{S_{kjt}}{C_j} TC_{kj} \quad \textbf{(2)}$$

In Eq (2), the first part denotes the cost incurred for purchasing the raw materials from suppliers, the second part shows transportation costs that are gained by multiplying the number of delivery events by transportation cost. The delivery frequency is the number of raw materials ordered divided by the capacity of transporting raw materials.

- Production cost is formulated as follows:

$$= \sum_i \sum_t \frac{Q_{it}}{B_i} PC_i \quad \textbf{(3)}$$

Eq (3) shows production cost is affected by the production cost and the number of products produced. The number of the batches produced is derived from the production frequency, namely the quantity of product produced divided by the batch size.

- Transportation cost is computed as follows:

$$= \sum_r \sum_i \sum_t \frac{S_{rit}}{C_i} TC_{ri} \quad \textbf{(4)}$$

Transportation cost (Eq (4)) is incurred by the manufacturer to deliver the plastic bag products produced to customers (MSMEs). The formulation is formulated using the delivery frequency and the cost of transportation. The frequency of delivery is gained by dividing the number of products delivered to MSMEs r for product i in time period t (Srit) with a single transport capacity of product i (Ci).

- Inventory cost is added up as follows:

$$= \sum_j I_{jt} IC_j + \sum_i I_{it} IC_i \quad \textbf{(5)}$$

The inventory cost includes the raw material inventory cost and the final product inventory cost (see Eq (5). Finally, the objective function of the integrated production and distribution planning model is presented in Eq (6), while the constraints are presented in Eq (7) – (17).

$$\text{Max Z} = \sum_r \sum_i \sum_t S_{rit} P_{ri} - (\sum_k \sum_j \sum_t S_{kjt} P_{kj} + \sum_k \sum_j \sum_t \frac{S_{kjt}}{C_j} TC_{kj} + \sum_i \sum_t \frac{Q_{it}}{B_i} PC_i + \sum_r \sum_i \sum_t \frac{S_{rit}}{C_i} TC_{ri} + \sum_j I_{jt} IC_j + \sum_i I_{it} IC_i) \quad \textbf{(6)}$$

Subject to:

Capacity constraints

$$S_{kjt} \le CP_{kj} \quad \forall k, j, t \quad \textbf{(7)}$$

$$\sum_i Q_{it} \le Cap \quad \forall t \quad \textbf{(8)}$$

Flow constraints

$Q_{it}W_{ij} \leq I_{jt-1} + \sum_k S_{kjt} \quad \forall i, j, t \neq 1$ **(9)**

$Q_{it}W_{ij} \leq \sum_k S_{kjt} \quad \forall i, j, t = 1$ **(10)**

$Q_{it} + I_{it} \geq \sum_r S_{rit} \quad \forall i, t = 1$ **(11)**

$Q_{it} + l_{it-1} \geq \sum_r S_{rit} \quad \forall i, t \neq 1$ **(12)**

$S_{rit} \leq D_{rit} \quad \forall r, i, t$ **(13)**

Inventory constraints

$I_{jt} = I_{jt-1} + \sum_k S_{kjt} - \sum_i Q_{it}W_{ij} \ \forall j, t \neq 1$ **(14)**

$I_{jt} = \sum_k S_{kjt} - \sum_i Q_{it}W_{ij} \quad \forall j, t = 1$ **(15)**

$I_{it} = I_{it-1} + Q_{it} - \sum_r S_{rit} \quad \forall i, t \neq 1$ **(16)**

$I_{it} = Q_{it} - \sum_r S_{rit} \quad \forall i, t = 1$ **(17)**

Eq (7) and (8) are the capacity constraints to ensure the delivered materials to the manufacturer will not exceed the delivery capacity of the suppliers and to prevent the production go over the production capacity.

Eq (9) to (13) express flow constraints of all supply chains. Eq (9) presents the number of products produced may not exceed the number of raw materials ordered. Eq (10) confirms the number of products produced based on the proportion of production will not exceed the raw materials that have been ordered from the supplier. Eq (11) states that the number of products delivered to MSMEs should not exceed the amount of production plus existing inventory in the period of t=1. Eq (12) ensures the number of products to be delivered should not exceed the amount of product produced plus the existing inventory in the previous period. Lastly, Eq (13) shows the demand fulfillment that expresses the production will not exceed the demand received by the manufacturer.

Eq (14) to (17) represent inventory balance constraints. Eq (14) represents raw material inventory in period t is the raw material left over from production plus inventory in the previous period. Eq (15) indicates raw material inventory in period 1 is raw material left over from production in that period. Eq (16) confirms product inventory in the manufacturer is the product that is not delivered yet and added by-products inventory in the previous period. Finally, Eq (17) shows product inventory in the manufacturer is the remaining products that are delivered to MSMEs.

Table 1. Transportation cost

Distance	Km	Tariffs per km
Close	>3-50	Rp. 6,500
Medium	>50-100	RP. 5,500
Far	>100	Rp. 5,000

Table 2. Vehicle information

Vehicle Data	Description
Fuel type	Ron82
Price (Rp/Litter)	Rp. 7,650
Distance/Litter	12.5 Km
Vehicle Capacity	1,000 Kg

Numerical Example

We provide the real problem for the numerical example. In this case, dealing with the integrated production and distribution decisions. As mentioned earlier, the proposed model is designed to deal with the decision on how many materials are to be delivered from suppliers to the manufacturer, how many products are to be produced, and how many products are to be delivered from the manufacturer to customers. All decisions are equipped with the date or period of the events.

Table 3. Processing time of the product

Processing Steps	Production Time (seconds)				
	Plastic Bag A	Plastic Bag B	Gaset A	Gaset B	Total
Preparation	57	52	57	52	218
Mixing	95	93	95	93	376
Hopper Stirring	90	75	90	75	330
Take up Winder	221	151	149	218	739
Blower	117	131	126	128	502
Grinding	465	473	460	462	1860
Total	1045	975	977	1028	4025

Table 4. Inventory related costs

Inventories	Costs
Material A (Rp/day)	205.88
Material B (Rp/day)	549.04
Product Roll (Rp/day)	192.16

Table 5. Material deliveries S_{kjt} (Kg)

KJ (*supplier*, materials)	t (period)		
	1	2	3
11	112,5	112,5	112,5
12	0	0	0
21	0	0	0
22	12,5	12,5	12,5

Table 6. Product produced Q_{it} (Kg)

i (product)	*t* (period)		
	1	2	3
1	0	0	45
2	0	0	10
3	70	50	70
4	55	75	0

Table 7. Product delivered S_{rit} (Kg) - example

Srit	17-1	17-2	17-3	17-4
1	0	0	0	0
2	0	0	0	0
3	10	0	10	0

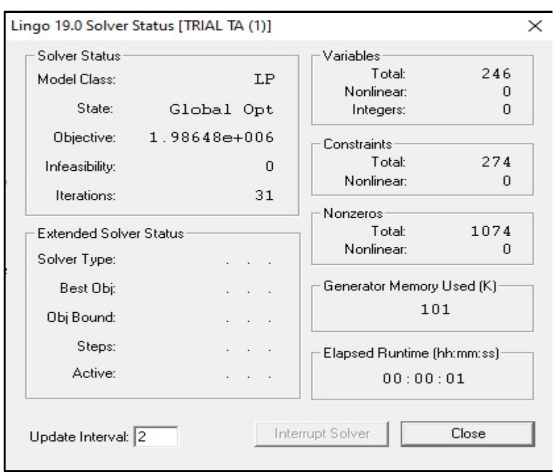

Figure 2. Lingo solver status

There are four types of products sold at the same price 125,000 Rp/roll, where each roll contains 5 kg of products. The demands are volatile and required for 17 customers (MSMEs) for 3 periods of planning. Table 1 and Table 2 show transportation costs related to distance and vehicle information respectively.

Materials are provided by two suppliers each supplier provides different materials. Supplier A supplies Polyethylene and it costs 17,500 Rp/Kg. While supplier B provides other material and it costs 25,000 Rp/Kg. Moreover, Table 3 presents the complete processing time of the product.

The production capacity is 7 hours/day which shows the manufacturer only runs its production a shift/day. Table 3 provides the processing time of the product in a roll where one roll is 5 Kg of products. Thus, the production capacity is 125 Kg/day. Moreover, the production capacity of a roll is Rp. 94,750 and inventory-related costs can be seen in Table 4.

Furthermore, to obtain the solutions from the above data, we use Lingo 19.0 Software. We provide a complete Lingo Code in Appendix 1. Figure 2 shows the Lingo solver status that indicates the solution gained is a global optimal.

Table 5 and Table 6 show the decisions on how many and when to deliver materials from suppliers to the manufacturer and the decisions on how many and when to produce the product respectively. While Table 7 shows the example of the product delivery schedule.

Sensitivity Analysis

We conduct sensitivity analysis to comprehend the model's behaviour by changing critical parameters and observing the changes to the objective function. The right behaviour shown by the objective function will validate the model.

Table 8 confirms that profit decreases when capacity is decreasing. Thus, it shows the validity of the model. We also inspect the decreasing capacity effects on decision variable Qit. The model decides to decrease Gaset production rather than plastic bag product (see Table 9). It is logical since plastic bag products provide more profit than gaset product.

IV. Conclusion

The model shows its ability to perform decision-making on materials delivery, product

Table 8. Production capacity sensitivity relative to the objective function

Changes	Capacity decrease	Profits / Objective Function	Percentage Profit Changes
1	0%	Rp 1.986.479	0
2	20%	Rp 1.604.909	-20%
3	54%	Rp 1.145.529	-54%
4	60%	Rp 821.770,1	-60%

Table 9. Production capacity sensitivity relative to the decision variable (Q_{it})

Capacity (Kg)	Capacity Decrease	Number of *i* product produced for 3 periods (Q_{it})			
		Plastic Bag A	Plastic Bag B	Gaset A	Gaset B
125	0%	45	10	190	130
100	20%	45	10	185	60
70	54%	45	10	115	40
50	60%	40	10	80	20

production, and product delivery. It simultaneously integrates production and distribution planning.

However, the model shows several points to improve such as zero delivery lead time, deterministic parameters, etc. The model might be improved by adding delivery lead time that may happen in real-life situations. A probabilistic or stochastic environment will improve the model complexity which is more suitable for more real-life situations.

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