

Development of Direction-Parallel Strategy for Shorting A Tool Path in The Triangular Pocket Machining

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Abstract. *The machining strategy is one of the parameters which practically influences the time of the different manufacturing geometric forms. The machining time directly relates to the machining efficiency of the tool paths. In area milling machining, there are two main types of tool path strategies: a direction-parallel milling and contour-parallel milling. Then direction-parallel milling is simple compared with a contour-parallel strategy. This paper proposes a new model of the direction-parallel machining strategy for triangular pockets to reduce the tool path length. The authors develop an analytical model by appending additional the tool path segments to the basis tool path for cutting un-machined area or scallops, which remained along the boundary. To validate its results, the researchers have compared them to the existing model found in the literature. For illustrating the computation of this model, the study includes two numerical examples. The results show that the proposed analytic direction-parallel model can reduce the total length of machining. Thus, it can take a shorter time for milling machining.*

Keywords: *analytical model, direction parallel, milling, tool path, triangular pocket machining, zigzag strategy.*

I. INTRODUCTION

One of the most crucial mechanical process in machining parts in computer-aided manufacturing (CAM) is pocket milling. Two-dimensional milling or pocket milling, i.e., cut a given two-dimensional region down to a fixed depth using a rapidly spinning circular milling tool, is the most common pocket machining problems (Kramer, 1992; Arya et al., 2001). The triangular pocket machining is frequently met in aerospace vehicle manufacturing because this machining supports achieving of structural properties, namely strong and light (Bieterman & Sandstrom, 2003). Two basic approaches of well-established tool path strategies for pocket milling are the direction parallel (zigzag or staircase) machining strategy and contour parallel (window-frame) machining strategy. These two tool path approaches are commonly applied in

the roughing phase as well as in the finishing phase (Choi & Jerard, 1998).

The direction-parallel strategy is preferably suited for face milling features. This strategy is much more straightforward concerning the computation, the ability to retain persistent chip loads in high-speed machining, and simple visualization (Sarma 1999; Park & Choi, 2001). However, in this machining strategy, the tool changes from up cut to down lead to the short lifetime the tool and the machine chatter. Besides, this method leaves some scallops on the wall of the pocket, so needs recurrent machining for any remaining regions.

While the ultimate goal of a machining strategy is to achieve a given final geometric shape within the shortest amount of time (Arkin et al., 2000; Kim & Choi, 2002), the cutting direction for the tool path generation is an essential variable in a staircase machining (Tang et al., 1998). Hence, this article proposes the new analytical model for extending the direction-parallel strategy, which proposed by Veeraamani and Gau (1997), in order to reduce the tool path length. In this research, the author focus on the case of triangular pocket machining.

The remainder of this paper is structured as follows. Firstly, the paper describes the research method. In this section, the author introduces related work to the research and development of the direction-parallel model. In the following

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section includes deriving the analytical model for the proposed tool path method of the direction-parallel strategy and numerical examples. For describing the computation of the proposed analytic model, the paper provides two numerical examples. The last section offers to conclude.

II. RESEARCH METHOD

Related Work

With a basic direction-parallel machining strategy, the tools move along line segments parallel to specified inclination in the alternating directions between adjacent paths. From Figure 1, there are two main segments in this strategy, i.e., the horizontal traversal segments (AB, CD, and so on) and the ascending segments (BC, DE, and so on).

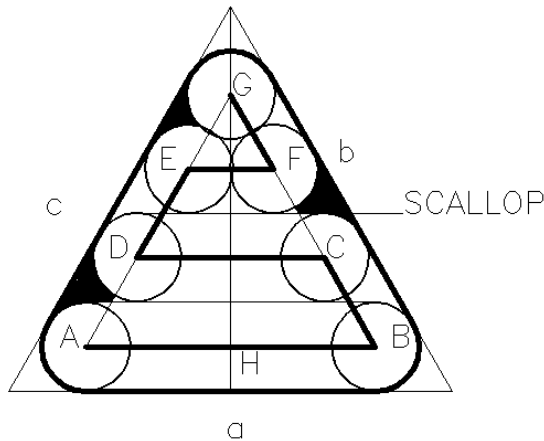


Figure 1. The illustrative tool path for basis direction-parallel machining.

Veeramani and Gau (1997) develop an analytical model for computing the total length of the cutting-tool path in inner portion stage (IPS). They argue that a direction-parallel strategy has a shorter tool path than a contour-parallel approach. In the IPS, the tool follows a zigzag machining strategy to machine the pocket from its basis to its top. When it has reached the top, it removes scallops from the other two sides by going around the boundary. This move is known as overlap. They employ a function $f(\alpha, \beta, \gamma, a, b, c, \rho, r)$ to describe the total tool-path length in IPS (Figure 2), where the notations of α , β , and γ represent the inner angles of the triangle

corresponding to the three sides a , b , and c , respectively (Figure 1). The ρ is the tool overlap coefficient (between 0 and 1) denoting the degree of overlap between two adjacent cutting paths. The r is the radius of the cutting tool.

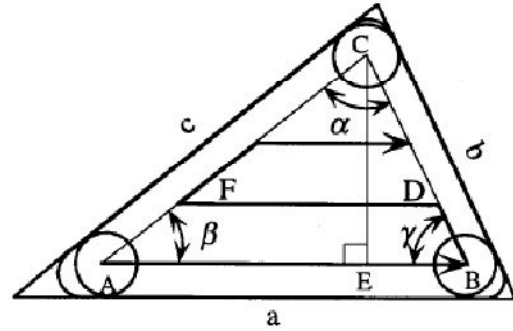


Figure 2. Illustration of IPS (Veeramani and Gau, 1997)

According to them, function f represents the sum of three segments, i.e., the horizontal segments, ascending segments, and the boundary segments. The analytical model corresponding to these three segments is given as follows.

The length of horizontal segments

$$\overline{AB} = a - r \cdot \cot\left(\frac{\beta}{2}\right) - r \cdot \cot\left(\frac{\gamma}{2}\right) = a - r \cdot \left(\cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\gamma}{2}\right) \right) \quad \dots(1)$$

$$\overline{DF} = \overline{AB} - 2 \cdot r \cdot \rho \cdot (\cot(\beta) + \cot(\gamma))$$

$$\overline{DF} = a - r \cdot \left(\cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\gamma}{2}\right) \right) - 2 \cdot r \cdot \rho \cdot (\cot(\beta) + \cot(\gamma)) \quad \dots(2)$$

If the number of horizontal or flat paths i cannot be zero then i is 1, 2, ..., n . The total of horizontal steps n in IPS can be calculated as

$$\frac{\sin(\beta)(c - r \cdot \cot(\alpha/2) - r \cdot \cot(\beta/2))}{(2 \cdot r \cdot \rho)} \quad \text{or} \quad \frac{\sin(\gamma)(c - r \cdot \cot(\alpha/2) - r \cdot \cot(\gamma/2))}{(2 \cdot r \cdot \rho)} \quad \dots(3)$$

Thus the total length of horizontal traversal paths is:

$$\sum_{i=1}^n \left[a - r \cdot \left(\cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\gamma}{2}\right) \right) - 2 \cdot r \cdot \rho \cdot (i-1) \cdot (\cot(\beta) + \cot(\gamma)) \right] \quad \dots(4)$$

The length of ascending segments are:

$$\overline{AC} = c - r \cdot \cot\left(\frac{\alpha}{2}\right) - r \cdot \cot\left(\frac{\beta}{2}\right) \quad \dots(5)$$

$$\overline{BC} = b - r \cdot \cot\left(\frac{\alpha}{2}\right) - r \cdot \cot\left(\frac{\gamma}{2}\right) \quad \dots(6)$$

The total length of the ascending parts is estimated as:

$$(\overline{AC} + \overline{BC})/2 = \left((b+c) - r \cdot \left(2\cot\left(\frac{\alpha}{2}\right) + \cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\gamma}{2}\right) \right) \right) / 2 \quad \dots(7)$$

The length of the boundary segments

$$(\overline{AB} + \overline{BC} + \overline{CA}) = (a+b+c) - 2r \cdot \left(\cot\left(\frac{\alpha}{2}\right) + \cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\gamma}{2}\right) \right) \quad \dots(8)$$

Thus, the total length of the tool-path in this stage is the sum of (4), (7), and (8).

The previous study (Chaeron, 2006) has shown that tool path of the direction-parallel strategy is longer than the contour-parallel plan. The direction-parallel strategy has a longer tool path because it needs the paths to around pocket for omitting scallops. This condition gives an idea to modify the basis direction-parallel strategy in which scallop removal can be done during the machining process.

Development of the new tool path model in the direction-parallel strategy

A description of this strategy is composed of three main steps (Figure 3). *First*, move inward offset with half of the tool diameter from the pocket boundary. The diameter of the tool is $2r$. The inward offset result is ABO triangle. *Second*, draw lines, which these are parallel to the longest side of the triangle. Because the machining process can be done without overlap, then the distance between the parallel lines is the same as the diameter of the tool (CE, FH, and so on). *Third*, draw tool path for eliminating scallop (DAE, GCH, and so on).

Due to a limitation that the corner radius of the pocket equals to the radius of the tool, so all the area of the pocket can be machined with only one tool diameter size. As shown in Figure 3, there are three main segments in this

development strategy:

1. The horizontal or flat paths (AB, CD, and so on) correspond to the parallel horizontal traversal segments.
2. The ascending paths (BC, AE, and so on) correspond to the ascending segments.
3. The connector paths (DA, GC, and so on) are used for eliminating the scallops along the pocket boundary.

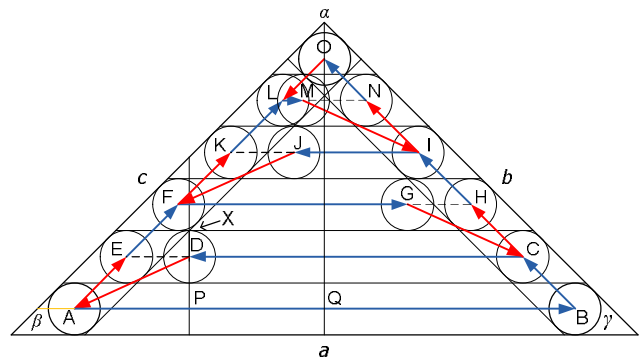


Figure 3. The illustrative tool path for the proposed direction-parallel machining

The first horizontal segment is done by machining from the left-end, using a cutting-tool with a diameter $2r$, to right-end along horizontal path AB. Next, the machining is executed at the ascending segments (BC) with an angle of γ . The second horizontal path is CD. Point D is the midpoint position of the tool in which it is perpendicular to point X. Point X is the intersection results of the tool peak with the inward result line offset of the AO line as far as r . Then, the un-machined area is eliminated by the machining process at the path DA and AE (Figure 4). The path DA directs it back to point A. After the cutting-tool is back to point A, it is done machining at the ascending paths, i.e., AE and EF, with an angle of β . Further, the determination of the tool path is done with the same sequence as above, until the whole triangular area undergoes the machining process.

III. RESULT AND DISCUSSION

Derivation of the proposed tool path model

The following assumptions are being made for developing the analytical model:

1. The machining strategy to be followed is the

- direction-parallel machining.
- The orientation of the tool-path is parallel to the longest edge of the pocket, and the tool-path begins at this edge.
 - The radius of the cutting is the same or smaller than the smallest corner radius of the pocket.

The length of the horizontal paths, which is parallel with a , is given by

$$\overline{AB} = a - r \cdot \cot\left(\frac{\beta}{2}\right) - r \cdot \cot\left(\frac{\gamma}{2}\right) = a - r \cdot \left(\cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\gamma}{2}\right) \right) \quad \dots(9)$$

$$\overline{CE} = \overline{AB} - (2 \cdot r - \rho) \cdot (\cot(\beta) + \cot(\gamma))$$

$$\overline{CE} = a - r \cdot \left(\cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\gamma}{2}\right) \right) - (2 \cdot r - \rho) \cdot (\cot(\beta) + \cot(\gamma)) \quad \dots(10)$$

From Figure 3, the length of line OQ can be calculated as

$$\overline{OQ} = \overline{AO} \sin(\beta) = \left(c - r \cdot \cot\left(\frac{\alpha}{2}\right) - r \cdot \cot\left(\frac{\beta}{2}\right) \right) \cdot \sin(\beta) \quad \dots(11)$$

The number of the horizontal paths (parallel lines to AB) i cannot be zero and the total of these steps n can be calculated as

$$n = \frac{\overline{IL}}{(2 \cdot r - \rho)} = \left\lceil \frac{\left(c - r \cdot \cot\left(\frac{\alpha}{2}\right) - r \cdot \cot\left(\frac{\beta}{2}\right) \right) \cdot \sin(\beta)}{(2 \cdot r - \rho)} \right\rceil \quad \dots(12)$$

The total length of the horizontal paths is given by

$$\sum_{i=1}^n \left[a - r \cdot \left(\cot\left(\frac{\beta}{2}\right) + \cot\left(\frac{\gamma}{2}\right) \right) - (2 \cdot r - \rho) \cdot (i-1) (\cot(\beta) + \cot(\gamma)) \right] \quad \dots(13)$$

For direction-parallel machining strategy, the ρ value is allowed zero.

The reduction of the length of the ascending or flat paths can be calculated by:

$$\overline{DE} = r \cdot \frac{(\cos(\beta)+1)}{\sin(\beta)}, \text{ for the odd paths } \dots(14a)$$

$$\overline{GH} = r \cdot \frac{(\cos(\gamma)+1)}{\sin(\gamma)}, \text{ for the even paths } \dots(14b)$$

where $n-1$ gives the number of reducing paths.

The total length of the ascending paths can be computed as

$$\overline{AI} + \overline{BI} = \left(c - r \cdot \cot\left(\frac{\alpha}{2}\right) - r \cdot \cot\left(\frac{\beta}{2}\right) + b - r \cdot \cot\left(\frac{\alpha}{2}\right) - r \cdot \cot\left(\frac{\beta}{2}\right) \right) \quad \dots(15)$$

The machining on the descending paths for removing the un-machined area undergoes the path DAE , GCH , and so forth. The length of these connection paths can be calculated as follows:

$$DA = 2 \cdot r \cdot \sqrt{\left(\frac{3 \cdot \cos(\beta) + 1}{2 \cdot \sin(\beta)} \right)^2 + 1} \quad \text{for the odd paths } \dots(16a)$$

$$GC = 2 \cdot r \cdot \sqrt{\left(\frac{3 \cdot \cos(\gamma) + 1}{2 \cdot \sin(\gamma)} \right)^2 + 1} \quad \text{for the even paths } \dots(16b)$$

where $n-1$ gives the number of the paths for the scallops removal.

Then, the total length of the tool paths TL can be represented by

$$TL = \text{Eq.(13)} - \text{Eq.(14)} + \text{Eq.(15)} + \text{Eq.(16)}$$

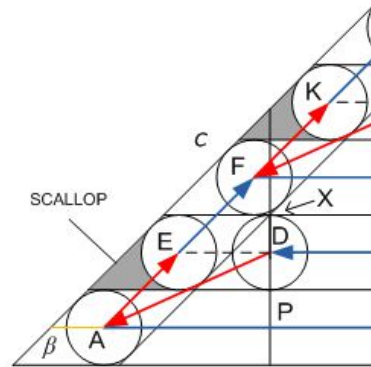


Figure 4. The illustrative tool paths for avoiding scallops

Numerical examples

To demonstrate the computation, the authors solve the following numerical example (Veeramani & Gau, 1997). First, an equilateral triangular pocket has angles $\alpha = \beta = \gamma = 60^\circ$, the sides length $a = b = c = 100$ mm, the curvature radius of the corners and the cutting-tool radius are 10 mm (Figure 5).

Second, a triangular pocket has angles $\alpha = 90^\circ$, $\beta = 53^\circ$, $\gamma = 37^\circ$ with the length of the sides $a = 100$ mm, $b = 80$ mm, $c = 60$ mm, the curvature radius of the corners and the radius of the cutting-tool are 5 mm (Figure 6).

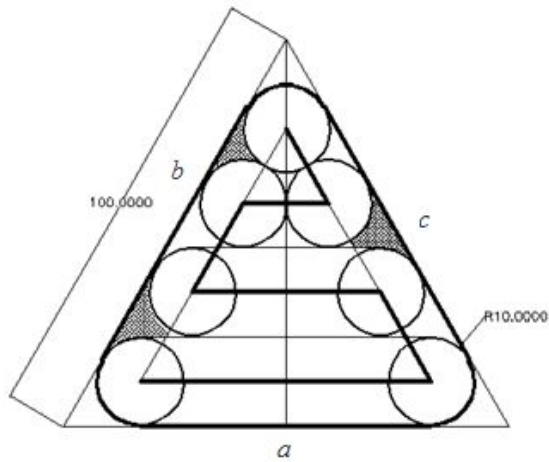


Figure 5. The illustrative tool path of the triangular pocket for the Example I

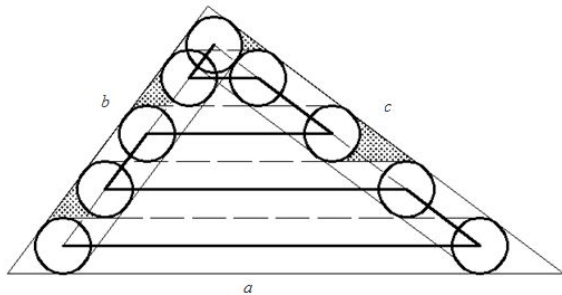


Figure 6. The illustrative tool path of the triangular pocket for Example II

In Figure 7 and 8, the illustrative tool paths using the proposed machining model is presented for the numerical Example I and II, respectively. The paper includes the computation for the examples, as follows.

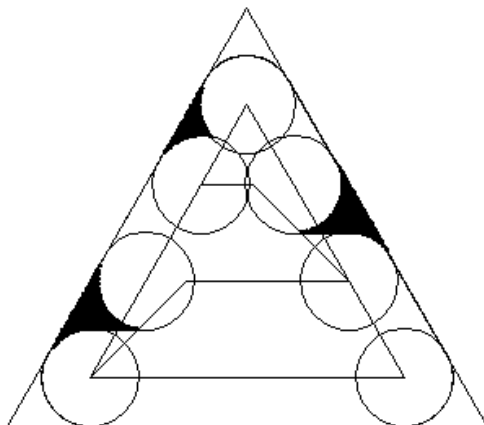


Figure 7. The illustrative tool path for the Example I using the proposed model

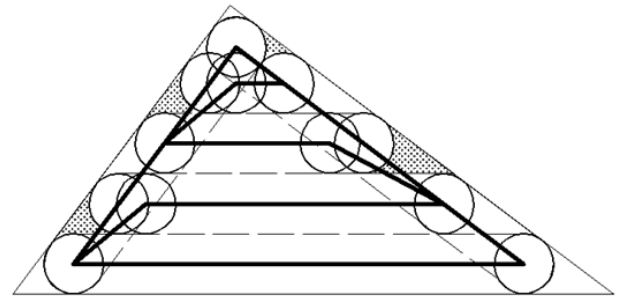


Figure 8. The illustrative tool path for Example II using the proposed model

Based on the Veeramani and Gau (1997) model. The total length of the parallel paths is calculated by Eq.(4).

$$n = \left[\frac{(100 - 10 \cdot \text{Cot}(30) - 10 \cdot \text{Cot}(30)) \cdot \text{Sin}(60)}{(2 \cdot 10 - 0)} \right] = 2.83 \approx 3$$

$i = 1, 2, 3$

$$= \sum_{i=1}^{2.83 \rightarrow 3} [100 - 34.641 - 20 \cdot 1.1547 \cdot (i - 1)]$$

$$= \sum_{i=1}^3 [65.359 - 23.094 \cdot (i - 1)]$$

$$= 126.795$$

The total length of the ascending paths is estimated by Eq.(7).

$$\frac{(100 + 100) - 4 \cdot 10 \cdot \text{Cot}30}{2} = \frac{200 - 69.282}{2} = 65.359$$

The length of the boundary paths for the removing scallops is calculated by Eq.(8).

$$(100 + 100 + 100) - 6 \cdot 10 \cdot \text{Cot}30$$

$$= 300 - 103.923 = 196.077$$

According to the proposed model, The total length of the horizontal segments is computed by Eq. (13).

$$n = \left[\frac{(100 - 10 \cdot \text{Cot}(30) - 10 \cdot \text{Cot}(30)) \cdot \text{Sin}(60)}{(2 \cdot 10 - 0)} \right] = 2.83 \approx 3$$

$i = 1, 2, 3$

$$\sum_{i=1}^n [100 - 10 \cdot (\text{Cot}(60/2) + \text{Cot}(60/2)) - (2 \cdot 10 - 0) \cdot (i - 1) \cdot (\text{Cot}60 + \text{Cot}60)]$$

Table 1. Comparison between two models of machining strategy

The total length of the segment (mm)	Veeramani and Gau (1997)		The proposed model	
	Example I	Example II	Example I	Example II
Horizontal (Reducing of the horizontal segment)	126.795	175.276	126.795 (-17.320)	175.276 (- 34.999)
Ascending Traversal (for eliminating scallops)	65.359 196.077	52.514 180.056	130.718 35.116	105.028 70.355
The total length of machining	388.231	407.846	275.309	315.660

$$= \sum_{i=1}^{283-3} [100-34641-20 \cdot 1.1547(i-1)]$$

$$= \sum_{i=1}^3 [65359-23094(i-1)]$$

$$= 126795$$

The reduction of the horizontal segments length can be calculated by Eq. (14). The number of reducing segments ($n-1$) is two, i.e., an odd path and an even path.

$$5 \cdot \frac{(\cos(60)+1)}{\sin(60)} = 8.660 \quad \text{for the odd path}$$

$$5 \cdot \frac{(\cos(60)+1)}{\sin(60)} = 8.660 \quad \text{for the even path}$$

The total length of the reduced horizontal paths is $8.660 + 8.660 = 17.320$

The total length of the ascending segments is obtained by Eq. (15)

$$(100 - 10 \cdot \cot(30)) - 10 \cdot \cot(30) + 100 - 10 \cdot \cot(30) - 10 \cdot \cot(30)$$

$$= 130.718$$

The length of the connection paths can be calculated by Eq.(16). The number of the paths for the scallops removal ($n-1$) is two, i.e., an odd path and an even path.

$$2 \cdot 5 \sqrt{\left(\frac{3 \cdot \cos(60)+1}{2 \cdot \sin(60)}\right)^2 + 1} = 17.558 \quad \text{for the odd path.}$$

$$2 \cdot 5 \sqrt{\left(\frac{3 \cdot \cos(60)+1}{2 \cdot \sin(60)}\right)^2 + 1} = 17.558 \quad \text{for the even path.}$$

Total length of the descending connection paths is $17.558 + 17.558 = 35.116$.

Using Eq. (11), the total of the horizontal steps n for the numerical Example II is four (with $i = 1, 2, 3, 4$). Consequently, the number of the reducing segments is three segments, i.e., the two odd paths and an even path. The number of paths for the scallops removal is three segments as well, i.e., the two odd paths and an even path.

The results summary of the calculation for the two numerical examples is shown in Table 1. This table shows the effect of changing the machining model on the triangular pocket for two cases.

IV. CONCLUSION

This article presents a new model of the tool path computation using an analytical method dedicated to the milling of the triangular pockets. Based on the results in two numerical examples, the proposed model gives the tool path length shorter than the existing model for all types of triangular pockets more than 22%. This result shows that there is the improvement of the pockets machining time. Further work will be done in order to compare the problem solution with contour-parallel machining strategy.

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