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Analysis of students' mathematical communication skills in web-based geometry in space and plane learning

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ABSTRACT

Mathematical communication skill in a mathematics learning process isa basic skill that must be possessed by students in developing the ability to think, reason, and solve problems. Especially in virtual learning, both online learning and blended learning, teachers must be able to conduct learning that built students' mathematical communication skills. Implementing web-based geometry in space and plane learning can be an alternative for forming mathematical communication skills with the aid of technology in today's virtual learning environment. This research aimed to examine 1) how web-based geometry in space and plane learning is implemented, 2) how students' mathematical communication skills are built through web-based geometry in space and plane learning, and 3) what learning alternatives need improvement to optimize students' mathematical communication skills. This research used a qualitative method with instruments in the form of lecturer activity observation sheet, student activity observation sheet, interview guide, and mathematical communication skills test. Data were collected through observation of learning implementation, mathematical communication skills testing, and interviews based on the test results. The data were analyzed with a descriptive technique. The results showed that 1) web-based geometry in space and field learning can be implemented by exploring with concept discussion on the activity pages, 2) web-based geometry in space and plane learning could help students explore ideas throughout the students' mathematical communication skills formation process, and 3) the learning could be improved by developing an HLT (hypothetical learning trajectory) using technology to develop a higher level of mathematical thinking ability.

INTRODUCTION

In the current COVID-19 pandemic situation, learning processes can be sustained virtually. Lecturers and students collaborate to enable effective learning by applying online learning and blended learning, which using the involvement of technology in the learning process(Anggraini & Mahmudi, 2021; Siron et al., 2020). In mathematics learning, technology use affects the instructions of mathematics concepts, thereby improving the implementation of mathematics learning and the

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activities involved (Forgasz, 2006). This can support the achievement of learning objectives and involve students in meaningful learning (Brantley-Dias & Ertmer, 2013; Muhtadi et al., 2018). Therefore, the use of technology is vital in the implementation of mathematics learning (Bakar et al., 2020). It indirectly supports learning implementation in the Industrial Revolution 4.0 era which requires that lecturers and students transfer newly formed knowledge online (Yuliardi et al., 2021).

Students studying mathematics education should pay more attention to the knowledge development process that takes place through online learning, particularly on the materials of similarity of triangles, congruence of triangles, area of rectangles, and area of triangles because these area prerequisite for understanding the following advanced geometry material. In addition, geometry in space and plane is also to be taught by students as prospective mathematics teachers at junior and senior high schools, which requires them to have a comprehensive understanding, both conceptually and procedurally (Muchlis et al., 2021). In fact, educational practice requires a space for students to act creatively in mathematics (Schoevers et al., 2019), which they feel can be created and done through learning geometry (Schoevers et al., 2020). The ability to solve geometric problems is important because it sets the first stepping stone to construct new mathematics knowledge (Levav-Waynberg & Leikin, 2012).

Geometry learning research obtained results, such as a) The object of research can be manipulated, and generalized concepts (Leung et al., 2013), and b) the use of animation media to think and interact in order (Baccaglini-Frank, 2019), to understanding of the geometric concepts. Furthermore, to form an understanding for the purpose of solving geometric problems, the ability, skill, and procedure to combine and develop concepts and ideas to form a deeper understanding of geometric concepts are necessary (Maarif et al., 2020; Schoevers et al., 2019; Warner et al., 2003). The process of conveying and analyzing ideas for one to be able to solve problems is a form of mathematical communication ability.

Mathematical communication is the process of expressing ideas by describing, analyzing, and informing on thoughts using numbers, pictures, tables, or words in mathematics learning (Morgan et al., 2014; NCTM, 2020; Yuniarti, 2016). It isa fundamental ability to be developed. According to the National Council of Teachers of Mathematics (NCTM, 2000), mathematical communication is the formation of understanding and monitoring of reasoning. Mathematical communication skills affect problem-solving abilities through the delivery of strategies and ideas (Fatimah, 2012; Pratiwi, 2015) while problem-solving abilities are among the skills needed in the 21st century (Erdoğan, 2019). Three indicators of mathematical communication are featured in this research, namely 1) consolidating thoughts (mathematical ideas or concepts) in writing, 2) interpreting mathematical ideas or concepts in the form of graphs, pictures, or tables, and 3) expressing mathematical ideas correctly (NCTM, 2000; Sumarmo, 2010).

In today's learning environment, it is critical to carry out online and blended learning while prioritizing students' abilities to be active and develop their thinking skills. Mathematical communication plays a basic role in the development of students' thinking skills, including understanding, reasoning, and problem-solving (Fatimah, 2012; NCTM, 2020; Pratiwi, 2015). However, some problems of students' low ability to express ideas or students' difficulties to interpret ideas in problem-solving are still present (Rizta & Antari, 2018; Susanti et al., 2019). .

Ng et al., (2020) stated that geometry understanding requires the experiences of visualization, reasoning, and the use of technology that can strengthen students' understanding of a concept. For this reason, new innovations of integrated technology are needed to support learning in the Industrial Revolution 4.0 and Society 5.0 era, which should be accessible by students in any place and at any time (Lane, 2016). To develop students' mathematical communication skills optimally and increase their desire to learn online, an appropriate technology-assisted learning model should be applied. Online learning should be designed using an innovative approach by considering the selection of the right platform so that it can improve students' competences (Mairing et al., 2021). The Internet and the web can be used as products of technological developments in learning to provide meaningful learning experiences (Fatahillah et al., 2020).

The use of the web as an online media in learning can help lecturers motivate students, encourage collaborative activities, improve students' thinking skills and procedural abilities, and develop students' creativity (Akcaoglu, 2016; Johnson & Onwuegbuzie, 2004). For learning purposes, the web can be designed according to the learning needs and combined with software use (Karagiannis et al., 2006). The use of the web in learning can provide texts, images, animations, and videos, examples and practice questions, and activities that stimulate students to carry out a series of thinking processes to generate in-depth knowledge (Derr et al., 2018; Hagerty et al., 2010). The ease offered by the web will facilitate interactive activities in learning.

Some relevant studies (Derr et al., 2018; Jonsdottir et al., 2017; Lyons et al., 2021; Pichon et al., 2021) concluded that web-based learning is an alternative learning approach that is effective at improving learning achievement, and further studies are suggested. However, not many studies have particularly examined the impact of web-based learning on mathematical communication skills. Therefore, based on the above considerations, we took a deep interest in developing a web-based learning media to optimize the formation of students' mathematical communication skills. The *first* objective of this study was to find out how web-based geometry in space and field learning is applied. *Second,* to find out how students' mathematical communication skills are formed through web-based geometry in space and field learning. *Third*, to find out what learning alternatives need to be improved to optimize students' mathematical communication skills. This research is critical to the attempt to gain a picture of how a learning process under the web-based geometri in space and plane learning is undertaken, thereby allowing the development of an appropriate learning model based on the findings. This research's results will also be useful for lecturers to improve students' mathematical communication skills through web-based geometry in space and plane learning. In addition, lecturers may also conduct better learning planning to avoid the problems encountered in this research.

METHODS

Research methods and participants

This research uses a qualitative approach. The *first* objective of this study is to provide an overview of the steps in geometry in the study of space and planes. *Second,* to describe students' problems when expressing and analyzing ideas in developing mathematical communication skills, to solve problems of similarity and conformity of triangles, rectangular area, and triangular area. *Third,* determine alternative learning improvements that use problem-based technology in the learning process and the results of student thinking analysis in web-based geometry in space and field learning. The research instruments used a mathematical communication skill test and interview guidelines that are in accordance with the test results and are supported by student and lecturer activity observation sheets. Before using the three instruments, expert validation was carried out on two mathematics education lecturers and a trial was conducted to get the reliability value of the questions of the mathematical communication ability test. The subjects of this research involved34 students who took the Geometry in Space and Plane course at the Mathematics Education Study Program, Faculty of Teacher Training and Education Science, at one of the state universities in Bengkulu province. They were then assigned to 7 different groups. Data were obtained by observing the geometry in space and plane learning implementation.

Data collection and data analysis

Data were gathered from mathematical communication skills test and interviews with students and the lecturer of the Geometry in Space and Plane course. The data were analyzed with a descriptive technique by analyzing synchronous learning records, analyzing student activities on the website, analyzing the results of the mathematical communication skills test, and analyzing the video record of the mathematical communication skills test interviews. The results of interviews and observations of learning activities were validated by one mathematics education lecturer. The limitation of this study is to analyze the mathematical communication skills of students who are the subjects of research and the research conducted only covers Bengkulu City. The procedure according to which this research was conducted is presented in Figure 1.

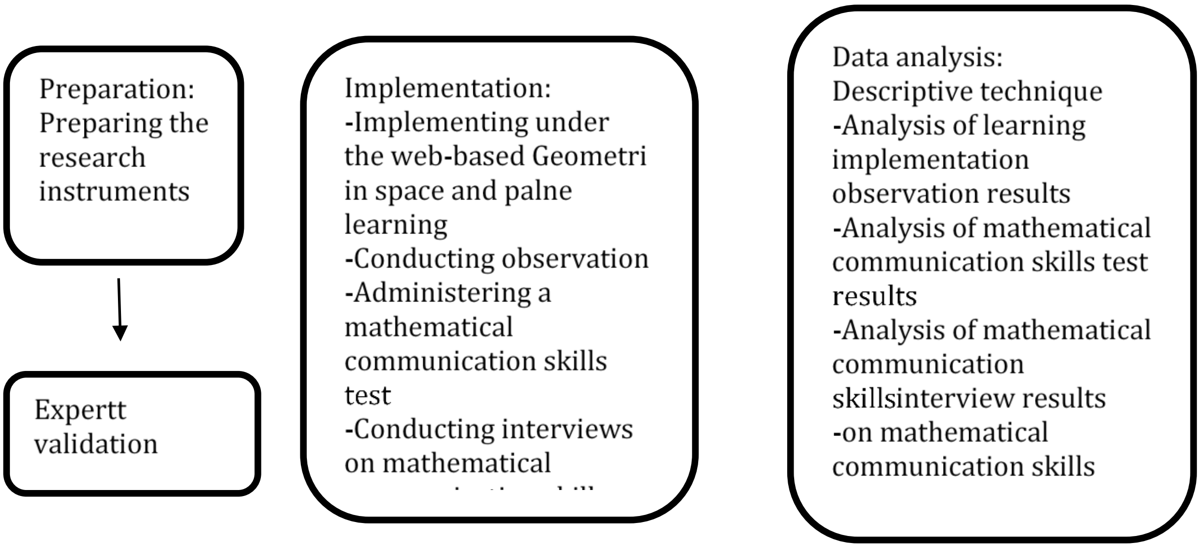


Figure 1. Research Procedure

FINDINGS

Application of web-based geometry in space and plane learning

Learning implementation took place from March to April 2021. Learning was carried out with GeoGebra software and a learning media in the form of a web application. Stages of learning process were based on Yuliardi et al., (2021) using an android application. The learning implementation process is described in Table 1.

The web-based geometry in space and plane learning media was developed in a student worksheet format. There isa page that guides students to carry out exploration activities using GeoGebra software to find concepts. This website has the capability of displaying mathematical symbols that are relevant to the topics during the learning process, such as the symbols of angles, exponents, and congruence. Students as users will be able to interact directly with the website after logging in. In addition, the website can directly display GeoGebra application to support exploration and concept discovery activities. There are several features available on the website, including video tutorials on using GeoGebra software online, an entry table to fill in the data from the output of the GeoGebra software exploration, and evaluation and chat features that can be used by students to discuss the concepts of similarity of triangles, the area of a quadrilateral, and the area of a triangle. The following isa display of the web-based geometry in space and plane learning media.

The web-based learning media development aimed to enable understanding and train students in expressing their mathematical ideas, developing them into concepts of similarity of triangles, the area of a quadrilateral, and the area of a triangle. The concept of the area of a quadrilateral covered by the website included the area of a square, the area of a rectangle, the area of a parallelogram, the area of a trapezoid, and the area of a kite. The web-based learning media was also designed to develop students' systematic and procedural skills in solving problems. With this web-based learning media, it was expected that students would be assisted in consolidating thoughts (mathematical ideas or concepts) in writing, interpreting mathematical ideas or concepts in the form of graphs, pictures, or tables, and expressing mathematical ideas appropriately.

Students' mathematical communication skills improvement through web-based geometry in space and plane learning

Based on the results of the mathematical communication skills test in the field, the achievement average of each indicator on the concepts of similarity of triangles, the area of a quadrilateral, and the area of a triangle is presented in Table 2.

The web-based geometry in space and plane learning media that had been developed was tested for its effectiveness. It was found that the average score of the students' mathematical

Table 1
The Web-Based Learning Implementation Steps

Apperception Step 1. The lecturer prepared students to start learning. 2. The students prepared learning resources and logged in to the website www.geometri.effie.info 3. The lecturer conveyed the learning objectives and explained the activities to be carried out on the website. 4. The lecturer made apperceptions about the topics of similarity of triangles, the area of a quadrilateral, and the area of a triangle.
Exploration and Elaboration Step 1. The lecturer explained the prerequisite knowledge as a basic concept to carry out activities to understand the topics of similarity of triangles, the area of a quadrilateral, and the area of a triangle. 2. The students explored the website, produced GeoGebra-assisted animations, and generalized concepts through the activities designed on the activity page. 3. The students completed some procedural questions; on a webpage there are boxes available for students to fill in their answers. 4. The students conducted a group discussion on the chat page. 5. The lecturer provided a guidance for students who faced difficulties. 6. The students were given an opportunity to present the results of the discussion through synchronous learning using Zoom application. 7. The students completed an evaluation at each meeting.
Confirmation and Closing Step The lecturer directed the students to provide conclusions and emphasize the truth of the concept or the procedure for finding and solving problems.

Table 2
Average Achievement of Each Mathematical Communication Skills Indicator

Mathematical Communication Skills Indicator	Achievement Average	Achievement of Mathematical Communication Skills
1. Consolidating thoughts (mathematical ideas or concepts) in writing	82.35% (28 of 34 student)	Students are able to write down what is known and asked from the question, and can express the method used to solve the problem.
2. Interpreting mathematical ideas or concepts in the form of graphs, pictures, or tables	79.41% (27 of 34 student)	Students can only do 2 different ways in the process of solving the same problem using pictures, graphics or narration.
3. Expressing mathematical ideas correctly	67.65% (23 of 34 students)	Students complete mathematical procedures correctly and correctly

communication skills was 80.09 and the level of learning completeness obtained was 73.53%. These scores reflect that the effective criteria were met, meaning that the web-based geometry in space and plane learning media was able to improve the students' mathematical communication skills. In addition, the students responded to the web-based geometry in space and plane learning media use through a questionnaire positively (88.24%).

The results of the students' mathematical communication skills test were analyzed to examine the students' thinking process and the obstacles that occurred during the learning implementation. The students were given the following problem:

From triangle MNP and triangle QRS, it is known that m∠M = m∠Q, m∠R = m∠N, m∠M = 56°, m∠R = 82°, $\underline{MN} = 9cm$, $\underline{QR} = 6cm$, $\underline{RS} = 7cm$, and $\underline{MP} = 12cm$. Determine m∠N and $\underline{NP}$.

To be able to solve this problem of similarity of triangles, the students needed to interpret the triangular figures. By paying attention to the side lengths and angles of the triangles in the problem, the students began to conduct an analysis to determine the concept to be applied in solving the problem. The answer from a student is provided in Figure 2.

As many as 28 students (82.35%) answered correctly, and the rest answered incorrectly. Errors occurred because some students could not determine the equivalent ratio of the triangles based on the corresponding sides of the triangles. As shown in Figure 3, the student was able to show that the triangles MNP and QRS were similar based on the corresponding sides and the congruent angles, suggesting that the student already had the basic idea. It was then found that m∠N = 82°. Meanwhile, the length of $\underline{NP}$ was calculated using the proportionof the lengths of the corresponding sides of the triangles, namely $\frac{\underline{NP}}{\underline{RS}} = \frac{\underline{NM}}{\underline{RQ}} \rightarrow \frac{\underline{NP}}{7} = \frac{9}{6} \rightarrow \underline{NP} = \frac{36}{6} \rightarrow \underline{NP} = 10.5\ cm$.

The next problem concerned a parallelogram. To work on that problem, an analysis of the parallelogram figure would be needed. The problem tested is as follows:

From parallelogram MNPQ, it is known that$\underline{QP}$ = 12 cm, $\underline{QM}$ = 9 cm, and the height of the parallelogram $\underline{QR}$ from point Q to the side $\underline{MN}$= 6 cm. Find the height of the parallelogram $\underline{QS}$ from point Q to the side $\underline{PN}$.

As many as 14 students (41.18%) answers are shown in Figure 4. The problem above is a parallelogram problem, which can be solved in two manners. *First*, it can be solved using the concept of similarity of triangles in several steps. However, students often interpret this type of problem using the concept of parallelogram with all known parts and then determining the length of the side to be calculated. In this example, they were able to determine the proportion of the side lengths of the two triangles found in the parallelogram, namely $\frac{\underline{QS}}{\underline{QR}} = \frac{\underline{QP}}{\underline{QM}}$. At this step, the students showed that they had understood the basic concepts of proportion and similarity of triangles. To determine the proportion of the sides of the triangles, it would be necessary to show first that the two triangles are similar. In Figure 4, the explanations given by the students were only based on the assumption that the triangles QRM and QSP are similar. The students whose answer is presented on the left section of the figure above tried to prove that the triangle QRM is similar to the triangle QSP, explaining m∠P = m∠M because they are opposite interior angles and $\underline{MQ} \sim \underline{PQ}$and ∠R = ∠S = 90°. After showing that the triangles QRM and QSP are similar, the next step was to solve the proportion for the triangles , namely: $\frac{\underline{QS}}{\underline{QR}} = \frac{\underline{QP}}{\underline{QM}} \Rightarrow \underline{QS} = \frac{\underline{QP}}{\underline{QM}} \,.\, \underline{QR} \Rightarrow \underline{QS} = \frac{12}{9} \,.\, 6 \Rightarrow \underline{QS} = 8\ cm$.

Second, this problem can be solved using the concept of the area of a parallelogram. From the results of the mathematical communication skills test for this problem, none of the students used this manner to solve the problem. If this concept is to be used, the problem can be solved as follows:

$$The\ area\ of\ MNPQ = \underline{MN}\ x\ \underline{QR} = \underline{NP}\ x\ \underline{QS}$$

$$\underline{MN}\ x\ \underline{QR} = \underline{NP}\ x\ \underline{QS}$$

$$12\ x\ 6 = 9\ x\ \underline{QS}$$

$$72 = 9\ x\ \underline{QS}$$

$$\underline{QS} = \frac{72}{9} = 8\ cm$$

The point to be considered in solving this problem is that students must understand how to interpret the formed shape so that they can apply the concept of the area of a parallelogram, the

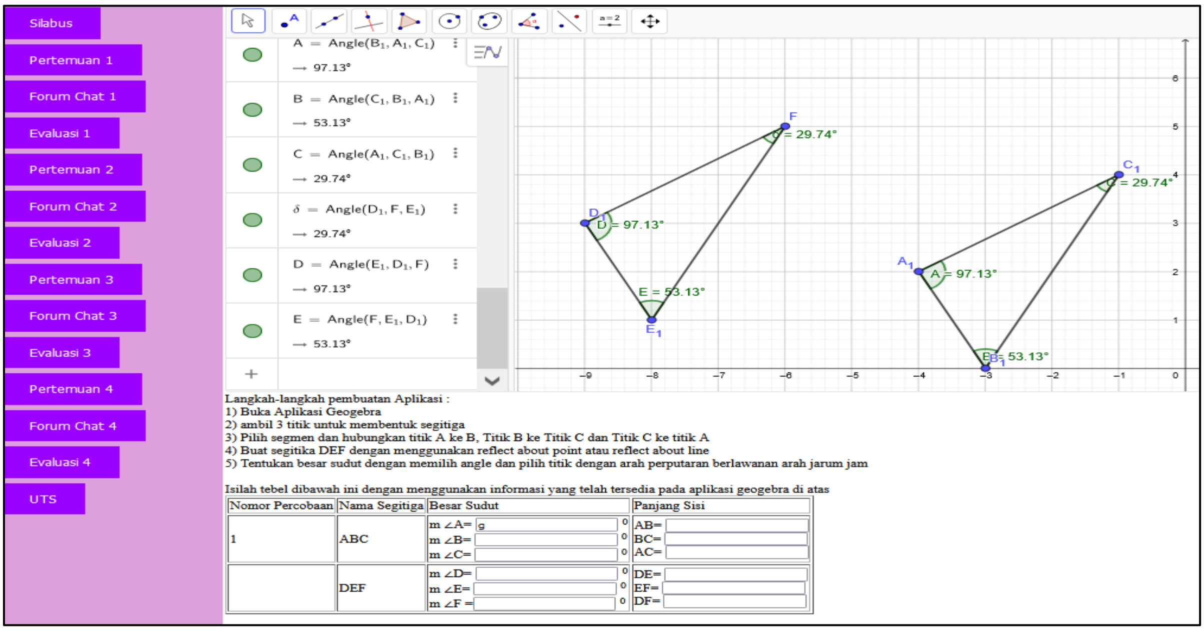


Figure 2. The Display of the Web-based Geometry in Space and Plane Learning Media

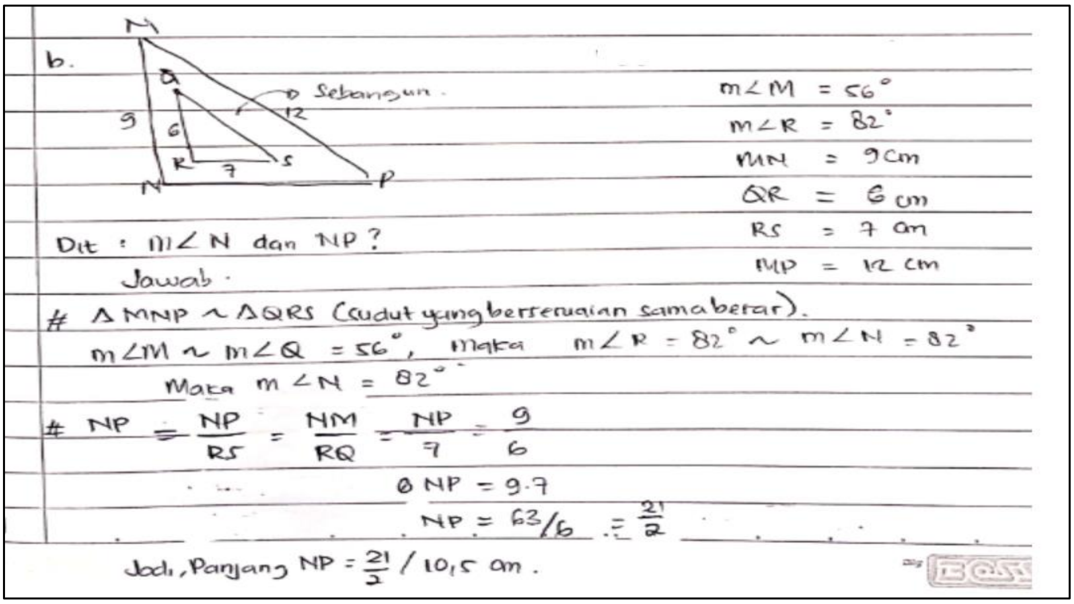


Figure 3. The Answer from a Student Using the Similarity of Triangles Concept

concept of similarity of triangles, or the concept of proportion to express mathematical ideas appropriately. The next problem was about a quadrilateral.
A quadrilateral RSTV has an area of 48 cm^2. Point X is on the side TV with TX:XV = 3:1, and point Y is on side ST with SY:YT = 1:3. Find the area of RYTX.

For the problem in Figure 5, the students might solve them by integrating the concepts of the area of a quadrilateral, the area of a triangle, and proportion. The answers of two students provided in Figure 5 show that they had been able to consolidate their thoughts by writing down the known parts first and then writing down their interpretation of the problem to which they wrote their solutions. They then interpreted the problem in the form of a rectangular figure (see Figure 5). The rectangular figure in the right section of Figure 5 is wrong because it depicts a three-dimensional shape, as shown by a dotted line in the back of the figure. Interviews with students find that the

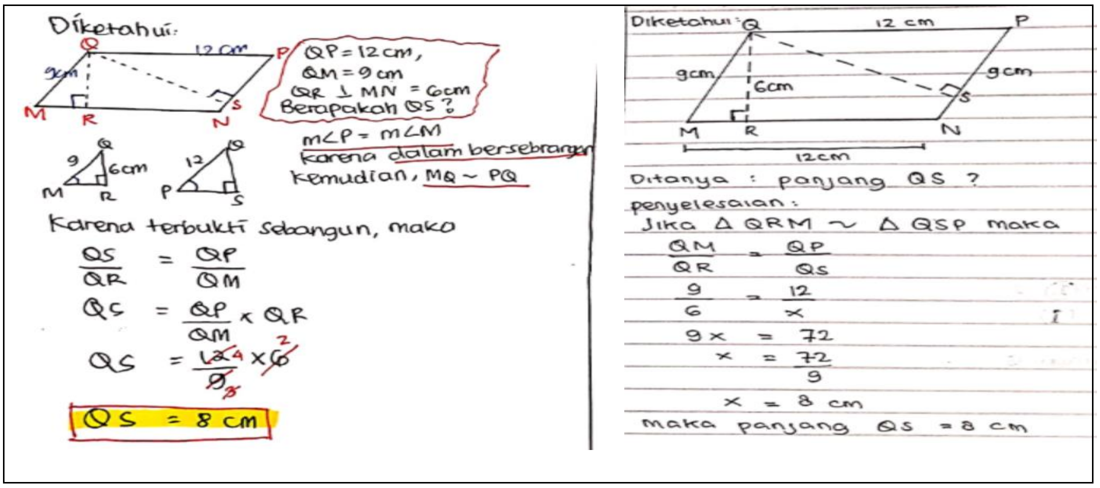


Figure 4. Students' Answers Using the Parallelogram Concept

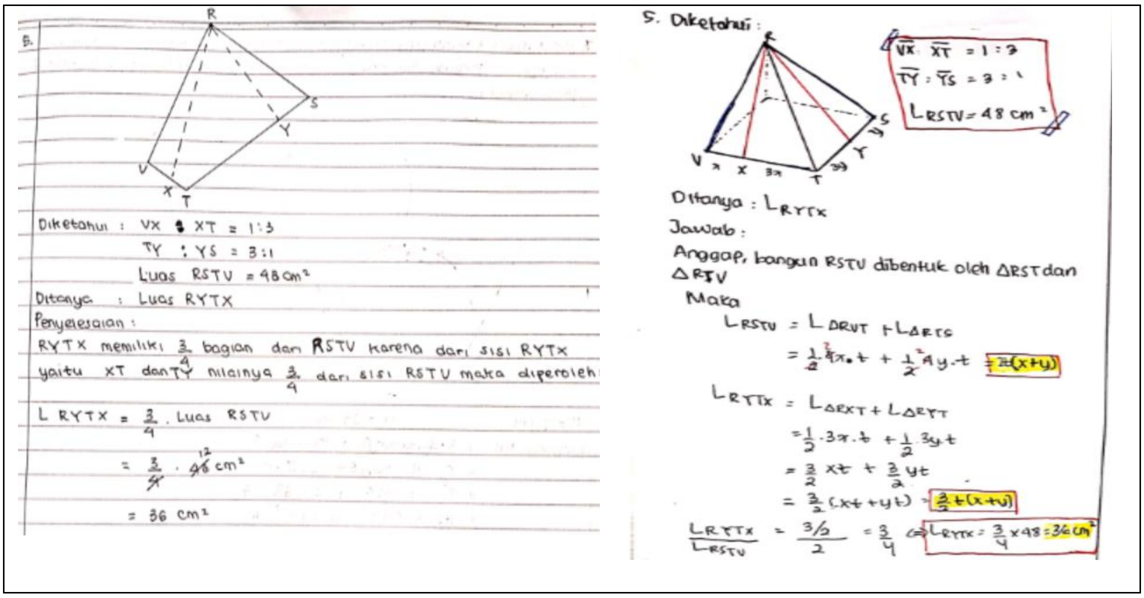
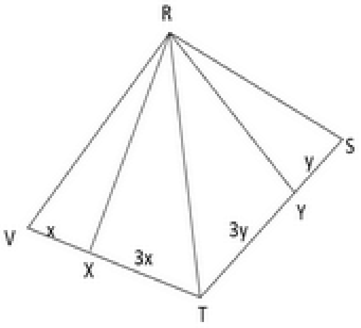


Figure 5. Students' Answers using the Quadrilateral Concept

addition of the dotted line is made because they want to find some other alternative planes that may be used to solve the quadrilateral problem. Although the final decision on the method used to determine the solution to the problem does not take advantage of the dotted made. The correct figure is the one on the left side. Then, they were asked to analyze the figure. In this problem, the students were asked to find out the relationship between the area of the shape for which the solution was to be determined and the shape whose area was known. The students had to make a connection of the concepts of the area of a quadrilateral, the area of a triangle, and proportionto find the solution. The student whose answer is shown in the left section of Figure 5 directly used the ratio of known side lengths to determine the area of the region to be solved, stating that the area of the region RYTX was ¾ of the area of the region RSTV. They argued that the lengths of the sides of RYTX, i.e., XT and TY, were ¾ of the lengths of the sides of RSTV. This shows that, in interpreting the figure, the student already had the idea that the area of RYTX = ¾ the area of RSTV. It therefore became necessary to demonstrate the truth of that idea. As can be seen on the left side of Figure 5, the student tried to demonstrate the truth by interpreting the figure formed as two triangles and assuming the length of each side using a variable, i.e., $\underline{VX} = x$, $\underline{XT} = 3x$, $\underline{TY} = 3y$, and$\underline{YS} = y$. The algebraic calculation procedure canbe seen below:



The figure RSTV is formed of triangle RST and triangle RSV. Therefore,
The area of RSTV = the area of triangle RVT + the area of triangle RTS
The area of RSTV = (½ . 4x . t)+ (½ . 4y . t)
The area of RSTV = 2t(x + y)
The area of RYTX = the area of triangle RXT + the area of triangle RYT
The area of RYTX = (½ . 3x . t) + (½ . 3y . t)
The area of RYTX = $\frac{3}{2}\ t(x + y)$

Then, the proportion of the area of RSTV to the area of RYTX is:

$$\frac{The\ area\ of\ RYTX}{The\ area\ of\ RSTV} = \frac{2t(x+y)}{\frac{3}{2}t(x+y)} = \frac{3}{4}$$

The area of RSTV = ¾ of the area of RYTX
Therefore, the area to be determined, RSTV, is calculated as follows: RSTV = ¾ . 48 cm^2 = 36 cm^2.

DISCUSSION

Based on the results of the mathematical communication skills test, observation, and interviews with students, one procedure that was often considered difficult by the students was to show the similarity of two triangles as a prerequisite for determining a proportion to find out the area of a quadrilateral. In addition, they were found to have mastered the basic concepts needed to solve the problem given, but they had difficulties to demonstrate the truth of their ideas using the concept of the area of a triangle. The problems found in students in solving mathematical communication skills problems in this quadrilateral material require attention from the lecturer. It is revealed by Confrey et al., (2017), Ivars et al., (2018),and Nickerson et al., (2017) that lecturers need to interpret and decide how to respond to students' mathematical ideas and analyze students' conceptual understanding, so that lecturers can develop learning activities that can build better ways of thinking, reasoning and communicating. In order to encourage students able to think in a better way, lecturers can use HLT (Hypothetical Learning Trajectory) which serves as a reference for lecturers in identifying learning objectives, interpreting students' mathematical thinking, and responding through appropriate instructions (Sztajn et al., 2012). It also applies in teaching and learning mathematics that uses technology.

The results of the interviews with students on the use of technology, in the form of a web-based learning platform, showed that the platform could increase the students' interest in studying geometry in space and plane because they were trained to be independent, responsible, and careful, learning was designed with a greater degree of structure, and the exploration with GeoGebra software allowed them to improve creativity and innovativeness. In addition, the technology-based learning media could improve the students' mathematical communication skills as indicated by their ability to represent problems in the form of figures and explore several ideas to solve the problems given. The students were able to choose, apply principles, and express ideas and mathematical concepts appropriately according to the concepts of similarity of triangles, the area of a quadrilateral, and the area of a triangle. The results of the error analysis at the time of the interviews showed that the students still experienced procedural errors in applying the concept of similarity of triangles and faced difficulties in proving the truth of the basic ideas they had constructed as the key to solving problems, such as the problems of parallelograms and quadrilaterals, in which case the students already formed ideas but experienced difficulties in finding the relationship between one concept and another concept.

This condition shows that learning implementation requires lecturers to encourage students to work together, engage in interesting mathematical activities, and establish a learning community (De Araujo et al., 2018; Dimmel & Herbst, 2017). Lecturers also need to help students to be able to communicate mathematical ideas effectively by practicing expressing mathematical ideas both orally and in writing, which will result in a better understanding of mathematical concepts, strengthened mathematical thinking ability, and better higher-order thinking ability(Fried, 2009; Liang, 2014). In addition, the results of the research conducted by (Sumarmo, 2010; Trenholm & Peschke, 2020) showed that the use of technology in the implementation of mathematics learning had good potential to be used to develop mathematical communication skills. Responding to students' conceptual errors , it is necessary to involve lecturers in identifying students' initial understanding and preparing possible actions that must be taken based on activities and responses given by students during learning. For this reason, it is necessary to prepare HLT (hypothetical learning trajectories) which can be used as guides for the development of students' mathematical thinking process in understanding concepts, making it easier for lecturers to analyze errors in students' mathematical thinking process as well as design alternative actions to address learning barriers that may occur in the learning process (Ivars et al., 2018; Nuraida & Amam, 2019; Simon, 2017).This is emphasized in the form of task design, community development in learning, and alternative instruction (De Araujo et al., 2018).

CONCLUSIONS

From the research results provided above, some conclusions can be drawn. *First,* Web-based plane and space geometry learning can streamline mathematical communication skills through exploring concepts on activity pages and carrying out discussions through web pages with an effectiveness value of 80,09. *Second,* the results of the analysis of the students' test answers showed that the students communicated their mathematical ideas in solving problems in various ways. This diversity illustrates the ways in which the students developed ideas and chose problem-solving strategies. In addition, several learning barriers were also found, namely procedural errors in applying concepts, lack of skills in communicating basic ideas as a prerequisite for forming solutions, and difficulties in showing the relationship between one concept and another. *Third,* technology-based learning implementation offers an alternative for developing students' mathematical communication skill. Lecturers' designed HLT scan serve as references to improve students' mathematical communication process in studying the concepts of similarity of triangles, the area of a quadrilateral, and the area of a triangle. These results ultimately lead to a suggestion to implement web-based learning under other learning models so that other mathematical thinking abilities can be improved according to the learning standards in higher education.

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