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A Ruppert's framework: How do prospective teachers develop analogical reasoning in solving algebraic problems?

Siti Lailiyah[1*], Kusaeri Kusaeri[1], Endah Retnowati[2], Erman Erman[3]

[1)]Department of Mathematics Ecuation, Universitas Islam Negeri Sunan Ampel Surabaya, Indonesia
[2)]Department of Mathematics Ecuation, State University of Yogyakarta, Indonesia
[3)]Department of Science Ecuation, State University of Surabaya, Indonesia

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ABSTRACT

It is widely agreed that knowing how prospective teachers develop analogical reasoning in solving problems is essential. Problem solving is specific domain that requires particular ways of analogical reasoning skill. The purposes of this study was to reveal the development of analogical reasoning and strategies used bya porspective teachers. The research design used a qualitative method. As many as sixty nine mathematics prospective teachers were involved voluntarily to complete algebraic tasks and 12 of them were interviewed to investigate their analogical reasoning and solution strategies. The data analysis used Ruppert's framework consisting of four components: structuring, mapping, applying, and verifying. It was found that the first three components were fully performed by the prospective teachers. However, the verifying stage was applied by prospective teachers in different ways. The dominant strategy used was a combined multiplication and addition. Their strategies varied according to the participants' general ability. The more strategies employed in solving problems, the better their analogical reasoning is becomes. This implies that instructional designs that will be developed by prospective teachers may vary. Therefore, during their candidature, they should be provided with many solving strategies in problem-solving to develop students' analogical reasoning.

INTRODUCTION

Analogical reasoning has been known as a very important aspect of learning, especially in understanding a concept (Amir-Mofidi et al., 2012; Kearney & Young, 2007; Vendetti et al., 2015). It is a basic process in solving everyday problems (Ayal et al., 2016; Lovett & Forbus, 2017; Meheus, 2000; Supratman, 2019), making conclusions (Holyoak, 2012), and training high-order thinking (Richland & Begolli, 2016; Richland & Simms, 2015). Many have agreed that it is a skill that students must have (Haglund et al., 2012; Magdas, 2015; Vendetti et al., 2015). Mostly has argued that it is related to students' cognitive and thinking abilities, especially higher-order thinking skills (Richland & Begolli, 2016; Richland & Simms, 2015). It is the ability to configure systems (Krzemien et al., 2017; Sternberg, 1977). The analogy is a basic process in everyday problem solving which refers to the process of comparing the source domain (existing problems) and the target domain (new problems) in terms of the similarities in the relationship between the items in each domain (English, 1993; Holyoak, 2012; Ruppert, 2013). Analogical reasoning is the process of adapting and adjusting previously learned old ways to solve new problems (Lailiyah et al., 2017; Vybihal & Shultz, 1989).

*Corresponding author: lailiyah@uinsby.ac.id

Sternberg (1977) develop six components of analogical reasoning, namely encoding, inferring, mapping, application, justification, and response. However, Ruppert (2013) simplifies these into four components: structuring, mapping, applying, and verifying. Structuring is the combination of encoding and inferring while verifying is the combination of justification and response. The choice of the Ruppert framework for analogical reasoning is because it are simpler and the use of Ruppert's perspective may help to analyze the stages of the development of prospective teachers' analogical reasoning in more accurately.

Structuring is the first step in identifying a mathematical object by coding its attributes in the source domain (a problem domain that has been obtained by students, will later form the basis for solving other similar problems), and making inferences from the relationships of object's attributes in the source (Gentner, 1983; Lailiyah et al., 2018). Ruppert (2013) states that the indicator of structuring is identifying each mathematical object that exists in the source problem by coding its attributes or characteristics and making inferences from identical relationships in the source problem (Richland & Simms, 2015).

Mapping is the second stage in examining identical relations and constructing identical conclusions in the source domain to be mapped and linked to the target domain (Gentner & Smith, 2012; Holyoak, 2012). Being able to do mapping means determining the identical relationship of the characteristics codes between the source problem and the target problem, making inferences from the similarity/identicality relationships of characteristics codes between the source problem and the target problem, and mapping the relationships obtained to the target problem (Gentner & Smith, 2012; Richland & Simms, 2015). The identification of attribute/characteristic codes is done by linking parts in the source problem and making inferences from identical relationships to all source problems (Indurkhya, 1991). The attribute/characteristic codes in question are the completion strategies used to determine the results of solving the above problem.

Applying is the third stage to apply identical relations from the source domain to solve problems in the target domain (Ruppert, 2013). The indicator of applying may be shown by the manipulating skill the relationships obtained from the source problem to the target problem to solve the target problem. The attribute/characteristic code in the mapping that is used to determine the result of solving the problem in the source problem is applied to the target problem so that the solution is obtained. Applying is the process of inserting new knowledge into the target domain based on the mapping (Kokinov & French, 2003). Applying as an extension of the mapping already established, thus adding new elements to the target in such a way that the mapping can be extended.

Verifying is the last stage to evaluate the strategy implemented and re-examine the relationships between domains (Ruppert, 2013). The step of verifying may involve algorithmic checking of the accuracy of the target problem's solution by tracing back the suitability of the target problem to the source problem (Donoghue, 2004). Verifying is the process of establishing the likelihood that the applying knowledge will turn out to apply to the target domain. Verifying is often implicit in the mechanisms of mapping and applying (Kokinov & French, 2003).

Analogical reasoning has been implemented in teaching mathematics. Algebra is one well-developed branch of mathematics and hence introduced in elementary school (Carraher et al., 2006; MacGregor, 2006; Stacey & Macgregor, 2006). In everyday life, algebra is used as a tool or a method to express relationships among variables, analyze and represent patterns, explore the nature of mathematics in various problem situations, and study mathematics at an advanced level (Kanbir et al., 2018; Lian & Idris, 2006). Besides, learning algebra is a practice to improve students' thinking ability (Cai et al., 2005; Carpenter et al., 2005). Students' thinking ability indicates their understanding of a concept (Kusaeri & Aditomo, 2019; Rifandi, 2017; Tan et al., 2020). Solving algebraic problems (target problems) also requires structural similarity/relational mapping of the problem in the source problem (English & Sharry, 1996). So, analogical reasoning is needed to solving algebraic problems because algebra concepts ask students to connect variables in the solution (English & Sharry, 1996; Lailiyah et al., 2018; Lian & Idris, 2006; Supratman, 2019). The use of analogical reasoning in learning mathematics material makes students easier to solve any problems by relating the material that has been studied to the previous material. The system of linear equations is one of the materials in algebra that is often encountered in everyday life and found in the mathematics curriculum in several countries (Cai et al., 2005).

The teacher's lack of attention to the analogical reasoning process in students may pose difficulty for students in learning algebra (English & Sharry, 1996; Masduki et al., 2017). This difficulty is often caused by their failure to understand algebraic concepts and algebraic notations in solving algebraic problems, thus resulting in the students having a low level of algebraic knowledge (Akgün & Özdemir, 2006; Feurzeig, 1986; Hoon et al., 2020). The difficulty of students in learning algebra was dominant is becaused 60% of algebra problems at each level in the mathematics curriculum required a high cognitive level. Besided that it is because of 32% a misconception of algebraic symbols and difficulties in mathematizing algebra (Jupri et al., 2014; Samo, 2008; Ubuz et al., 2010).

Students' analogical reasoning abilities are at medium and low levels when solving algebra were not high and students experienced difficulties or made errors in completing analogy tasks (Glady et al., 2017; Lailiyah et al., 2017; Morrison et al., 2010; Lindsey Richland et al., 2006). In case of low-level ability in students' reasoning might be caused that students had difficulty to focus on the question's information (Alexander et al., 1987; Glady et al., 2017; Harrison & De Jong, 2005; Leon & Revelle, 1985). However, many attempts have been made to improve students' analogical reasoning such as through the development of learning strategies, learning methods, and analogy tasks (Christie, 2020; Maarif, 2016; Peled, 2007). The development of analogical reasoning with analogy task according to (Sternberg & Rifkin, 1979) consists of 5 ways: 1) Availability of Component Operations, 2) Strategy for Combining Multiple Component Operations, 3) Strategy for Combining Multiple Executions of a Single Component Operation, 4) Consistency in Use of strategy, and 5) Component-Operation Latencies and Errors.

The analogical reasoning of prospective teachers can be developed in various ways, one of which is developing increasing and demanding tasks and their solving strategies (Antal, 2004). Identification of the level of prospective teachers' analogical reasoning levels (high, medium, and low) can be done by identifying the number of strategies used in solving these problems. Many solving strategies are concerned with finding the correct solution to the analogy task (Whitely & Barnes, 1979). The more strategies are used in completing an algebraic task, the higher the analogical reasoning level. By describing the analogical reasoning of prospective mathematics teachers in solving algebraic problems, educators can develop various learning methods so that the analogical reasoning is increases.

Teachers' analogical reasoning can increase their pedagogical and teaching practice skills (Mozzer & Justi, 2013). In addition, their analogical reasoning skills can also make learning more effective (Gust & Kühnberger, 2006). Therefore, they should be learned when referring to is a prospective teacher. It is the prospective teacher who will teach the material to students; if the prospective teacher has a good analogical reasoning ability, he will be able to develop his knowledge well (Alexander et al., 1987; Casakin & Goldschmidt, 2000). In addition, prospective teachers who have good analogical reasoning skills can support the learning process, such as being able to ask the right question that stimulates the thinking abilities of their students (van den Kieboom et al., 2014; Walkoe, 2014). Therefore, it is important to analyze the analogical reasoning of prospective teachers.

Research on analogical reasoning on teachers or prospective teachers has been extensive (English, 2004; Goswami; Usha C, 2012; Goswami, 2013; Haglund et al., 2012; Krzemien et al., 2017; Morrison et al., 2010; Richland et al., 2006). The research by (Mozzer & Justi, 2013) examines the analogical reasoning of chemistry teachers, while (Antal, 2004) examines how to develop analogical reasoning of biology teachers, and (Agustina et al., 2020) examine the analogical reasoning abilities of prospective mathematics teachers on calculus problems. Thus, the present study will discuss how prospective mathematics teachers developed analogical reasoning in completing analogy tasks in algebra. In addition, the analogical reasoning under study is associated with the strategies used in solving it. The purpose of this study is to reveal in detail how prospective mathematics teachers developed analogical reasoning, particularly in algebraic problem-solving.

The development of analogical reasoning for prospective teachers in this study is very important, because it can develop various learning strategies in the classroom. If a teacher has good analogical reasoning, he or she can develop various types of math tasks, either the complexity of the task or the variety of contexts used. Due to the importance of analogical reasoning for a teacher, it is necessary to do more research in this area. This study examines analogical reasoning of prospective teachers in soving algebra based on the Ruppert framework, and how to develop analogical reasoning for prospective teachers in solving algebra problems.

METHODS

Research design and participants

This research method is qualitative research, the purposes of this study was to reveal in detail analogical reasoning of prospective teachers in solving algebra problem. This research uses the Miles et al., (2014) qualitative research method. Qualitative research is multimethod research with a naturalistic approach that begins with identifying, classifying, and summarizing the subject matter, attempting to make sense of, or interpret, phenomena in terms of the meanings people bring to them (Aspers & Corte, 2019). Participants were asked to complete the task by writing down the problem solutions in detail. The research is designed to identify, classify, and summarize the results. A total of sixty nine mathematics prospective teachers who filled out the research instrument voluntarily were selected by purposive sampling. Taking the participant using a purposive sampling technique is because the participant in this study is a prospective teacher so the chosen referring to are education students in the final semester. The mathematics prospective teacher education program is an educational program that produces professional, innovative, and Islamic-characterized mathematics education graduates, through a learning process that combines research and development of creative, innovative, and Islamic mathematics education. The courses taken by prospective teachers are pure mathematics, learning mathematics in schools, and learning Islamic mathematics. The reason for taking the participant is only for prospective teachers in the 7th to 13th semesters (final semester prospective teachers) because the final semester prospective teachers have got all the math materials. These mathematics prospective teachers had the foundation skill in algebra because they had already taken the algebra courses

Instruments and data collection

The instruments in this study were algebra tasks and interview guides. The algebra task aimed to explore participants' analogical reasoning in completing algebra tasks, while the interview guide was aimed to explore their analogical reasoning process as they were doing the algebra tasks. Data triangulation used in this study were technical and source triangulations.

There were two stages of research data collection, namely tests and interviews. The researchers obtained the picture of the analogical reasoning process of mathematics prospective teachers through the test, where prospective teachers have never been taught analogical reasoning of the Ruppert model in completing the task on this research instrument before. In interviews the researchers attempted to reveal why they used certain strategies in their analogical reasoning stages. The task given was an algebraic operation problem about an analogical reasoning test modified from Indurkhya's analogical reasoning problems (Indurkhya, 1991). Modification of the problem lies in the material. Indurkhya's analogical reasoning problems use problems in geometry, while disagreement this study use problems in algebra. The instrument in this study was validated by 5 people, namely 1 professor of applied mathematics, 1 lecturer of applied mathematics, 1 lecturer of mathematics education, and 2 high school mathematics teachers.

The validation of this instrument is directed at he suitability of the problem with the research objectives, the construction of the problem, and the suitability of the language used. The assessment criteria of this instrument are that it can explore the ability of analogical reasoning, can use the source of the problem in solving the target problem, and can find out the relationship between the source of the problem and the target problem. There are four criteria for assessing problem construction in this instrument, first, the problems given can lead to the process of analogical reasoning. Second, the problem boundaries given are sufficient to solve the problem and are clear. Third, a given source problem forms a certain code. Fourth, the code generated in the source problem can be applied to the target problem. There are five criteria for language assessment, first using language with good and correct Indonesian rules. Second, the problem formulation uses words that students know, Third, does not use the local language. Fourth, the limitations and formulation of the problem do not lead to multiple interpretations. Fifth, communicative problem formulation.

Table 1
Indicator of analogical reasoning components

No.	Analogical Reasoning Component	Indicator
1.	Structuring	a. identify each algebraic object within the algebra task. b. make inferences from the relations of identical algebraic objects in all source problems.
2.	Mapping	a. determine the identical algebraic object relationships between the source problem and the target problem. b. make inferences from the algebraic code identicality relationships between the source problem and the target problem. c. map the relationships obtained onto the target problem.
3.	Applying	apply the algebraic code obtained from relationships between the source problem to the target problem to solve the target problem.
4.	Verifying	double-check the accuracy of the target problem's solution by checking the suitability of the target problem to the source problem.

Determine the x and y values (in various ways) based on the following three figures! Give your reasons!

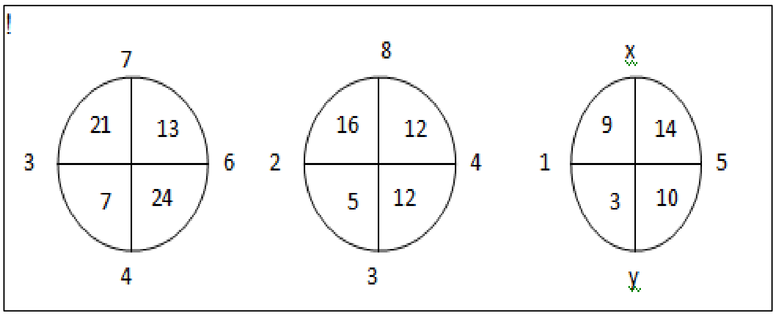


Figure 1. An algebra problem to observe analogical reasoning

Based on the results of the validation test, the instrument developed by the researcher was declared good. There were several improvements and suggestions, first, the problem with the initial instrument was a linear equation of one variable, it was suggested "to replace the system of equations with two variables". Second, the problem on the research instrument only consists of one question. It is recommended that "the question needs to be added, not just one question, is it enough to represent algebra?". However, the researcher adds to the limitation of this research that the mathematical problem in this study is an algebraic problem with the material of a system of two-variable linear equations. Third, the writing of "How to get the value of ..." is recommended to be replaced with "Determine the value of ..." or "Find a value....". Fourth, comments in this study: "Research conducted in class with material that affects prospective teachers or prospective teachers are enthusiastic about doing it". Next, the researcher revised the instrument based on input from the validators.

The problem can be seen in Figure 1. The algebra task was given by google classroom and participants were asked to complete them at home. When the participants were working on the task, there was no supervision from the researchers so that participants could complete the task independently with their strategies. The time given to complete the task was one day. The completed task was also submitted by google classroom.

Figure 1 is an algebra problem that contains analogical reasoning. Problems containing the source problem (in the first and second circles) and the target problem (in the third circle). The problem solving is performed by mapping and applying the inferences from the attributes/characteristics code relationships in the source problem to the target problem. The identification of attribute/characteristic codes is done by linking parts in the source problem and making inferences from identical relationships to all source problems (Indurkhya, 1991). The attribute/characteristic codes in question are the completion strategies used to determine the results of solving the problem above. The outlines of the task were structuring, mapping, applying, and verifying (Table 1). There are 3 categories in the analogical reasoning component: competent, less competent, and incompetent. It is said to be competent if each indicator is carried out correctly. The category is less competent if each indicator is carried out, but it is not correct. While the category is not competent if each indicator is not carried out. The indicators for each stage of the analogical reasoning components are presented below.

The interview, which involved 12 prospective mathematics teachers selected using purposive sampling based on their quality of the reasoning in completing the algebra task, was conducted via video calls. The twelve prospective teachers selected were as follows: one participant completed the task in four ways, two participants completed the task in three ways, three participants completed the task in two ways, and six participants singly completed task. The interview outline was structured based on the four aspects of analogical reasoning, namely structuring, mapping, applying, and verifying. Participants were given questions in the order of questions written in the interview guide. Researchers could adjust the questions in the interview guide according to the circumstances and answers of the participants. The interview was video-recorded to make sure nothing was missed (Cresswell, 2005).

Data analysis

The data was analyzed with data reduction, data presentation, conclusion drawing and verification (Matthew B Miles & Huberman, 1994). Data reduction refers to the process of selecting, simplifying, abstracting, and transforming the data that appear in written-up field notes or transcriptions. The interview transcripts were condensed by selecting, focusing, and simplifying the interview data by making abstractions, by classifying interview results based on the analogical reasoning components. In data reduction, each of the authors coded students' answers and interview transcripts in a deductive way (Table 1). The consistency of students' analogical reasoning was also examined using the data, classifying the quality of the reasoning in completing the algebra task to determine the development of analogical reasoning. The data display is an organized, compressed assembly of information that permits conclusion drawing and action. We analyzed the results of the coding to get a consensus. In data display, we organized, a compressed assembly of students' answers in matrix. The researchers verified the truthfulness of the transcripts by listening to the interview audio again to eliminate transcription errors, classify, identify, and analyze the participants' analogical reasoning. In conclusion drawing and verification, as the matrix fills up, preliminary conclusions are drawn and verified. Based on the matrix, we concluded each student's analogical reasoning. Drawing conclusions are based on the components category of analogical reasoning. Meanwhile, the quality of student answers is based on the completion strategy used.

FINDINGS

The results of analyzing the participants' completed algebra tasks show that their analogical reasoning only covered the structuring, mapping and applying components. Meanwhile, the verifying component did not appear in their completed work. The participants' skills in the structuring, mapping, and applying components were within the 'competent' category. Nevertheless, the verifying component was discovered only when the 12 selected participants were interviewed. When interviewed, the 12 participants stated that they double-checked their answers. Some double-checked their answers by substituting the values obtained into the source problem, some by rewriting referring to on another answer sheet, and some by looking at the consistency of their answers from the various methods they used. Based on the interviews of the 12 participants, it was be expected that the other participants also did the verifying process without writing it on their answer sheets.

The results of participants' analogical reasoning show that they could identify each algebraic object in the algebra task correctly. All participants noticed that the sections in the circles share the same pattern. Furthermore, they identified algebraic objects in the source problem, namely the first circle and the second circle, which is illustrated with the work of a participant as presented in Figure

Table 2
The participant's procedure in doing structuring

Steps	Interview excerpt
1. identifying each algebraic object within the algebra task.	*... I saw that there were three circles ... I identified the number pattern in the first circle, and after that I compared it to the second circle, and it turned out that the pattern was the same.*
2. drawing inferences from algebraic object relationships that are identical in all source problems	*Another method that I used was addition, multiplication, and division of the numbers in the first and second circles, which were then applied to the third circle.*

Table 3
The participant's procedure in doing mapping

Steps	Interview excerpt
1. determining identical algebraic object relations between the source problem and the target problem.	*Yes, the two circles share the same pattern.*
2. drawing inferences from the algebraic code identical relationships between the source problem and the target problem.	*First I tried to find the relationships between circle one and circle two. ... Then circle one's pattern is like that, circle two's pattern is the same, thus the third circle must have the same pattern.*
3. mapping the acquired relationships to the target problem	*I thought the x and y values of the third circle would be obtained by applying the same operations as those in circle one and circle two*

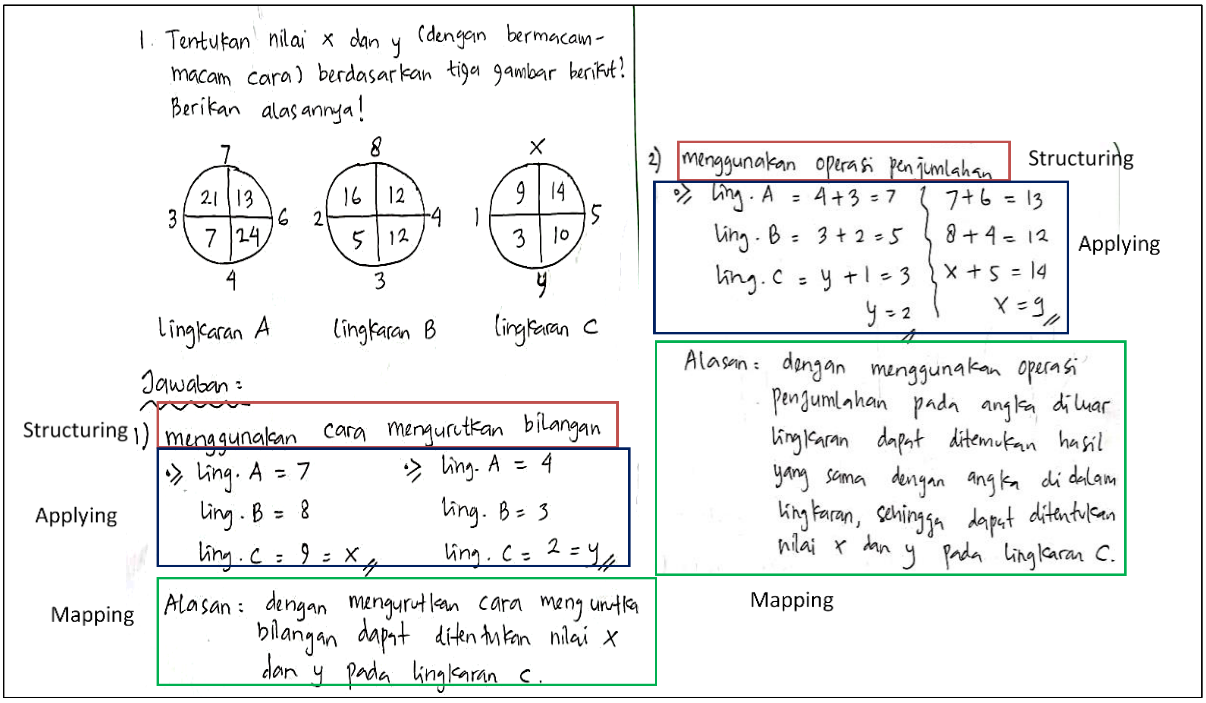
2 of the structuring code. Meanwhile, the characteristic codes obtained varied, including addition, multiplication, division, subtraction, number patterns, and a combination of several operations. The participant's procedure for doing structuring is presented in Table 2.

The participants' analogical reasoning shows that they were able to determine the identical learning object relationship between the source problem and the target problem and map it to the target problem. All participants mapped the same relationships between the first and second circles to the third circle as shown in Figure 2 code mapping. The participant's procedure for doing mapping is presented in Table 3.

The participants' analogical reasoning shows that they were able to apply the algebraic objects' relationships obtained from the source problem to the target problem in solving the target problem. The strategy used by most of the participants was a combination of addition to getting the x value and multiplication to get the y value (54 participants). Other strategies used by the participants were number patterns (14 participants), addition (5 participants), multiplication (3 participants), division (4 participants), division to obtain y, and subtraction to obtain x (5 participants). These strategies are shown in Figure 2 Applying the code. The participant's procedure in applying is presented in Table 4.

The participants' analogical reasoning shows that they did not carry out verification. However, based on the interview results, 12 participants did the verifying stage. The 12 participants double-checked the accuracy of the target problem's solution by checking the suitability of the target roblem to the source problem. Some participants verified their answers on scrap paper; some saw the similarity of the answers in the various ways used, and the remaining participants substituted the x and y values they obtained into the questions. The participants' verifying procedure is shown in Table 5.

Based on the results of the above research, it can be concluded that the analogical reasoning of mathematics prospective teachers has satisfied all the indicators of analogical reasoning components. All participants were able to identify each algebraic object in the algebra task and make inferences from the relations of algebraic objects that are identical in the source problem. They were



Translate in English:
Determine the x and y values (in various ways) based on the following three figures! Give your reasons!
Answer:
1. *Using method by sorting numbers*
   Circle A = 7 Circle A = 4
   Circle B = 8 Circle B = 3
   Circle C = 9 = x Circle C = 2 = y
   Reason:
   By using numbers we can determine how to sort the x and y values at circle C.

2) Using the addition operation
Circle A = 4+3 = 7 7+6 = 13
Circle B = 3+2 = 5 8+4 = 12
Circle C = y+1 = 3 x+5 =14
y=2 x= 9
Reason:
Using the addition operation on the numbers outside the circle, we can find the same results with the number in the cir
so that we can determine the values of x and y in circle C.

Figure. 2 The written answer of participant 1

able to determine identical algebraic object relationships between the source problem and the target problem, draw inferences from the algebraic codes' identical relationship between the source problem and the target problem, and map the resulting relationships to the target problem. They were also able to apply algebraic code relationships derived from the source problem to the target problem to solve the target problem. All participants were able to double-check the accuracy of the target problem's solution by checking the suitability of the target problem to the source problem. So, the prospective teachers' analogical reasoning ini completing an algebraic task fulfilled all the components of analogical reasoning, namely structuring, mapping, applying, and verifying in the "competent" category.

The obstacle faced by most of the participants was that they did not carefully read the task instructions. The task was to determine the x and y values in different ways, but most of them obtained using one solution or strategy only. Strategies are the number of certain ways or techniques in solving a problem. There are 5 strategies for solving problems in analogical reasoning: 1)

Table 4
The participant's procedure in doing applying

Steps	**Interview excerpt**
applying the algebraic code relationships obtained from the source problem to the target problem in solving the target problem	*... Yes, the same as circles one and two. Because of this, we are asked to determine x and y based on the following three pictures. This means that these three pictures must be related. So, in picture one and picture two, the relationship is the same pattern. So in the third picture, I think the pattern must be the same. So I decided to use the same pattern to define the third picture.*

Table 5
The participant's procedure in doing verifying

Steps	Interview excerpt
Double-checking the accuracy of the target problem's solution by checking the suitability of the target problem to the source problem	*How to check: there were two answers, x is 9 and y is 2. I substituted it for the third circle, so the x above is 9 and the y becomes 2. We checked it using the pattern on circle one and circle two whether these numbers match the pattern or not. It turned out it matched, such as 3 is obtained from* $1 + y$ *which means* $1 + 2 = 3$ *is correct. Then, 10 is obtained from* 5×2*. Then, 9 is* 1×9*. Then the 14 is obtained from* $9 + 4$*. So with the previous pattern, it turns out that x and y along with these other numbers are the true values*

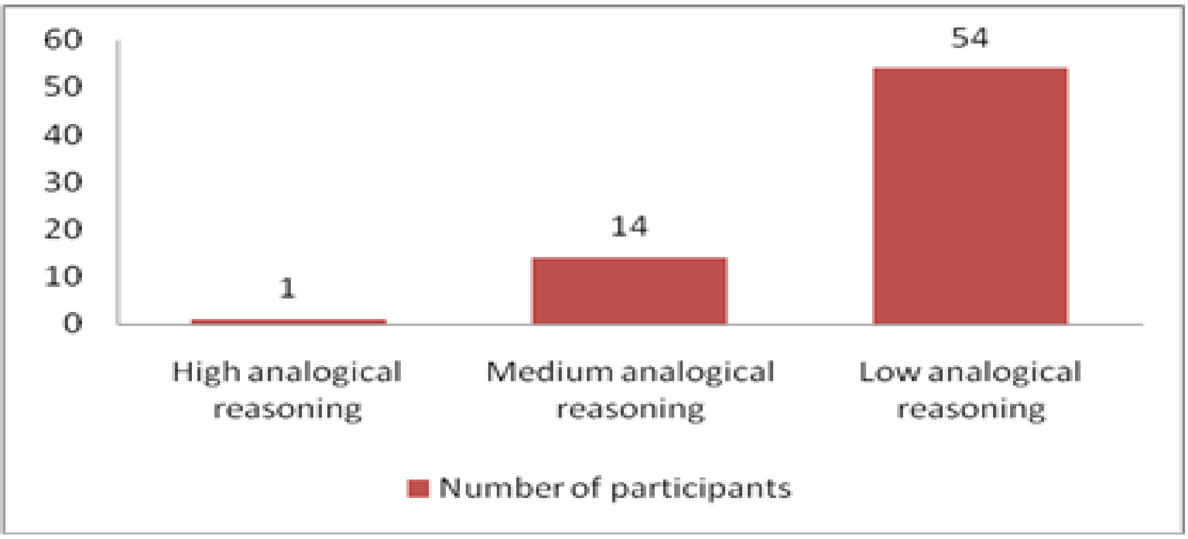


Figure. 3 Analogical reasoning level of all participants

availability of component operations, 2) strategy for combining multiple component operations, 3) strategy for combining multiple executions of a single component operation, 4) consistency in the use of strategy, and 5) component-operation latencies and errors. The dominant strategy used by most participants was multiplication operation to determine the y value and addition to determine the x value. This shows that most of the prospective teachers had difficulty in developing their analogical reasoning.

Furthermore, analysis of the participants' works in completing the algebraic tasks indicate various completion strategies in developing analogical reasoning. Fivety four participants used one completion strategy only, 12 participants used two completion strategies, two participants used three completion strategies, and one participant used four completion strategies. The strategies used by these participants demonstrate their analogical reasoning abilities. An example of various strategies used by a participant, code-named T1 (a participant who used four completion strategies), is presented in Table 6.

Table 6

An example of T1's various solution strategies in completing the analogical task

No.	Type of Strategy	Analogical Reasoning Component	Description and Analysis of Participant's Work and Interview Results
1.	strategy for combining multiple executions of a single component operation	Structuring	T1 labeled circle one as circle A, circle two as circle B, and the last circle as circle C (as in Figure 1). T1 began sorting the numbers outside circle A and circle B. From identifying circle A and circle B, T1 inferred that the numbers outside circle A and circle B formed an arithmetic pattern with a difference of 1 (d = 1).
		Mapping	After drawing inference on circles A and B (the source problems), T1 mapped it to circle C (the target problem).
		Applying	The number pattern with d = 1 was applied to circle C and it was found that the values of x = 9 and y = 2.
		Verifying	In this first method, T1 did not double-check the answer.
2.	availability of component operations	Structuring	T1 likened the locations of the numbers outside the circles to the cardinal directions (north, east, south, and west), and the numbers inside the circle with quadrant positions (quadrant I, quadrant II, quadrant III, and quadrant IV). T1 added the numbers outside Circle A (north and east) which are equal to the number inside the circle (quadrant I). Then, T1 added the numbers outside the circle (west and south) which is equal to the number inside the circle (quadrant III). This is also applicable to circle B. Therefore, T1 concluded that the sum of two numbers outside the circle was equal to a number inside the circle.
		Mapping	Next, T1 mapped that the inference to circle C would be the same as the inference from circles A and B.
		Applying	T1 added the numbers outside the circle and it was found that x = 9 and y = 2.
		Verifying	Because the answers to x and y from the first and second circles are the same, T1 was sure of his answers.
3.	strategy for combining multiple component operations	Structuring	T1 multiplied the numbers outside circle A (north and west) and the result was the number inside the circle (quadrant II); similarly, the multiplication of numbers outside the circle (east and south) is equal to the number inside circle A (quadrant IV). The same is true for circle B. T1 concluded that the third potential strategy to obtain the x and y values was multiplication.
		Mapping	Next, T1 mapped his inference to circle C.
		Applying	T1 did multiplication outside the circle and the result was the same as the number in C, so the values of x = 9 and y = 6 were obtained.
		Verifying	T1 compared the answers obtained from the various strategies he used.
4.	consistency in the use of strategy	Structuring	T1 divided the number inside circle A (quadrant II) by a number outside the circle (west), and the result was the same as a number outside the circle (north). Likewise, the number in circle A (quadrant IV) was divided by the number inside the circle (east), and the result was the same as the number in the circle (south). The same was true for circle B. Therefore, T1 inferred that the next step was division.
		Mapping	Next, T1 mapped the inference from circles A and B to circle C.
		Applying	T1 executed the inference he obtained to circle C so that the values of x = 9 and y = 2 were obtained.
		Verifying	T1 compared the answers obtained from the various strategies he had used and calculated them again on another answer sheet.

Table 7
Analogical reasoning level based on the completion strategy

No.	Number of completion strategies used	Analogical reasoning level
1.	≥ 4 completion strategies	High
2.	2 to 3 completion strategies	Medium
3.	1 completion strategy	Low

In Table 6, it appears that participants who completed the analogy task with various strategies had good analogical reasoning. All components of analogical reasoning, namely structuring, mapping, applying, and verifying were within the "competent" category. In addition to using various strategies in completing the task, the participants also checked the answers on another sheet of paper to ensure that their answers were correct.

Based on the results of this research, it can be inferred that the level of prospective teachers' analogical reasoning can be identified by knowing their completion strategies. The use of numerous strategies indicates that participants have developed analogical reasoning and their answers to the analogy task are correct. The following is an overview of the relationships between the completion strategy and analogical reasoning level (Table 7).

Table 7 describes analogical reasoning levels. If the number of completion strategies used in solving problems is more than four, the analogical reasoning level will be high. If the number of completion strategies used in solving problems is two to three, the level of reasoning analogy is medium. Meanwhile, if there is only one strategy to solve it, the analogical reasoning level will be low. A description of the participants' analogical reasoning level in this study based on the completion strategies they used is presented in Figure 3.

Figure 3 describes the analogical reasoning levels of all participants in this study. One participant is in the high level of analogical reasoning; 14 participants are in the medium level of analogical reasoning; 54 participants are in the low level of analogical reasoning. Therefore, it can be inferred that the analogical reasoning level of the prospective teachers in this study was low.

The development of analogical reasoning in solving algebraic problems can be done by developing many problem-solving strategies. The more strategies used, the higher-order thinking and good analogical reasoning are required. The problem-solving strategies developed by students include number patterns, addition, multiplication, and division. The strategy that is widely used by students in solving the algebra problem is the number patterns strategy.

DISCUSSION

Based on the results of the analysis, it is known that all prospective teachers in this study fulfill all components of analogical reasoning (structuring, mapping, applying, and verifying). They has written answer work that shows only 3 components (structuring, mapping, and applying), while the verifying component can be known based on the results of the interview. This is by (Koichu et al., 2013) that 62.5% of teachers can solve problems well while the remaining teachers have not been able to solve them well.

Based on the results of structuring of analogical reasoning in prospective teachers, it was found that the operating pattern that applies to the numbers contained in the first and second source problems. They described this identification in a written form on the answer sheet. Then, they were able to look for identical relationships between the source problem and the target problem, and draw inferences from the similarity/identicality of the characteristic codes between the source problem and the target problem. After that, the identified relationships were mapped onto the target problem. (Holyoak, 2012) view that analogical reasoning can be used as a basis for drawing inferences based on the similarities between the source and target domains.

Next, the prospective teachers does mapping that applies to the source problems and extrapolates the patterns to the target problem. After finding the operation patterns, the prospective teachers applied the operation patterns to the target problem. The prospective teachers conducted calculations coherently and clearly. This is by (Ruppert, 2013) observation that the applying stage is the moment to apply identical relations from the source domain to solve the problem in the target domain.

Next, after getting the values requested by the task, the prospective teachers doing verified with checking the accuracy of their answers on another sheet of paper. Based on this interview, it is possible that other prospective teachers accustomed to solving problems also did the same thing, namely checking the accuracy of the answers they had obtained. (Terlouw & Pilot, 1990) that getting used to solving problems will have an impact on one's self-regulation in solving a problem. The self-regulation in question is double-checking the correctness of the answers a person has obtained.

The analogical reasoning of the prospective mathematics teacher in this study is included in the competent category and fulfills the Ruppert framework indicator. It is appropriate analogical reasoning is a fundamental aspect of a person's cognition, either teacher or student. Teachers who have good analogical reasoning have good cognitive abilities (Vamvakoussi, 2019). With a good mastery of analogical reasoning, it will be useful for teaching mathematics. Analogical reasoning can also be used as a link in various materials in mathematics, such as from numbers to geometry, from geometry to algebra, and others (Vamvakoussi, 2017). It aims to make it easier for students to understand mathematical material.

Analogical reasoning is part of higher-order thinking which involves certain mental or cognitive processes that focus on and control the thinking patterns. Analogical reasoning can train higher-order thinking skills (HOTS in teachers and students alike (Richland & Begolli, 2016; Richland & Simms, 2015). Therefore, prospective teachers must practice their own analogical reasoning. In addition, students must be trained in analogical reasoning so that their higher-order thinking skills can improve.

This study seeks to answer various completion strategies used by prospective mathematics teachers in developing analogical reasoning, especially in solving an algebraic problem. The data show that the prospective teachers identified the patterns of operations applicable to the numbers in the first and second source problems. The number of completion strategies used by the prospective teachers varied: some used four methods, some three methods, some two methods, and some only used one method. The last type, as the interview later revealed, only knew one particular method and had never seen the alternatives. This means that prospective teachers can make use of various strategies and analogical reasoning if they learn to solve problems with various solving strategies. The strategies used by the prospective teachers in solving this algebraic analogical reasoning task were based on their experience in solving similar problems in the past. This is by (Phye, 1992) view that an effective problem-solving way is applying strategies by taking or remembering formal procedures previously studied to solve the problem at hand.

The prospective teachers, in completing the algebra analogy task, used four strategies. This shows that the use of numerous strategies indicates the participants' analogical reasoning skills and the accuracy of their answers to these problems. This is in line with (Whitely & Barnes, 1979) observation that the number of completion strategies used is related to obtaining the correct solution for the analogy task. Consequently, a lecturer should teach various strategies for solving math problems to understand the concepts and various completion strategies. In addition, this can help prospective teachers in their future work as teachers. They will be able to develop various mathematical problems, which ultimately will develop good analogical reasoning for their students. This is consistent with the (Murray & Macdonald, 1997) that lecturers' knowledge and teaching abilities contribute directly to students' knowledge.

In addition to numerous strategies used to identify prospective teachers' analogical reasoning ability, another way to develop analogical reasoning skills is developing a task that is interesting and demands higher levels of thinking (tasks that are unprocedural. Prospective teachers who have analogical reasoning skills will be able to complete the task and develop their High Order Thinking Skills (HOTS). HOTS is an effective assessment to measure students' development and achievement holistically (Wilson & Narasuman, 2020). The task can be given after the lesson ends or integrated into a textbook or worksheet. (Antal, 2004) that prospective teachers' analogical reasoning can be developed by giving increasing and demanding tasks.

Mathematical problems that are good for analogical reasoning are those that contain an element of giving justifications. This encourages students to explicitly write the reasons for completing the task in the way they do so that the mapping component within the analogical reasoning can be observed by the teacher. In addition, if the math problem contains a warning "Make sure your answers are correct", then it is likely that students will write the verification component explicitly on the answer paper, as stated by one of the participants that he checked the answers on another sheet of paper.

Analogical reasoning is ability can be determined based on how the math problem is given. Therefore, prospective teachers must design mathematical problems in various way that improves analogical reasoning. The design of a mathematical problem that can nurture analogical reasoning is using unprocedural questions or open-ended questions, and instructs the students to "justify your answers" and "make sure your answers are correct".

The development of various strategies requires a considerable amount of learning time, thus in giving math problems, prospective teachers do not need to impose time limits on students when solving these problems. In addition, as analogical reasoning requires processing, it should be studied not only within one field. Therefore, prospective teachers can create problems in fields other than mathematics. The development of analogical reasoning with analogy tasks should be applied as often as possible so that students get used to and introduce students to analogical reasoning.

CONCLUSIONS

The prospective teachers' analogical reasoning in completing an algebraic task fulfilled all the components of analogical reasoning, namely structuring, mapping, applying, and verifying, and it was under the "competent" category. Mostly, the fourth component of analogical reasoning, namely verifying, did not appear in the prospective teachers' written answers but emerged during the interview process. The prospective teachers did the verification by checking on another sheet of paper; some did it by substituting the answers obtained into questions, and the others by comparing the answers from various methods or the completion strategies used.

The development of analogical reasoning in solving algebraic problems can be done by developing many solving strategies and an interesting task (unprocedural task). Another way to develop it is to develop task that require higher-order thinking. The more solving strategies are used, the better is the analogical reasoning skills. Prospective teachers who have analogical reasoning skills will be able to complete the task and be able to develop their high-order thinking skills.

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