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Identifying Students' Errors on Fractions

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Abstract. *Many studies have revealed that fraction is a complicated mathematics topic for students. Students struggle to solve problems including comparison and addition of fractions correctly. However, some students often make some common mistakes in solving the mathematics problems. There are three types of errors in solving mathematics problems, which are factual error, procedural error, and computational error. This study was aimed at investigating various mistakes by students in problems related to fractions. A set of validated problems about comparing and adding fractions was assigned to third grade students of SD N Laboratorium Unesa Surabaya. The results indicated that some students were not aware on how to compare and add fractions. Most of these students employed incorrect strategies categorized as procedural and conceptual errors.*

Keywords: *errors, fraction, factual error, procedural error, computational error*

Introduction

The aim of learning mathematics is building students' knowledge of mathematical concepts. It is essential for teachers to carry out assessment in order to figure out to what extent students comprehend a given topic. Understanding students' mistakes in solving mathematical problems gives an insight for teachers about what causes might affect those mistakes and solutions to avoid such mistakes in the future.

Fraction, as revealed by some studies, is considered as challenging topic for students (e.g. Hasemann, 1981; Streefland, 1991; Cramer et al., 2002; van Galen et al., 2008; Charalambous & Pitta-Pantazi, 2007). Many studies suggested some error patterns in some topics in fractions such as interpretations of fractions, comparing fractions, and addition and subtracting fractions. Students have tendency to use rule-based procedure in solving fractions problems without understanding the problems (Howard, 1991).

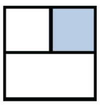


Figure 1. An example of students' mistake on fractions

As an example, as shown by McNamara and Shaughnessy (2011), when students are asked to write the fractions of given shaded area beside, the students answered 1/3. Students already know that the numerator represents the number of shaded parts and the denominator represents the unshaded parts. However, the student couldnot notice that the parts are in different sizes. These mistakes by students might be because teachers stress more on formal algorithms than on bulding students' understanding and reasoning (Idris and Narayanan, 2011).

Students' Mathematical Errors

In solving mathematics problems, both conceptual and procedural knowledge are needed. Referring to Morales' study (2014, p. 1), conceptual knowledge refers to "*knowledge of math facts and properties that are recognized as being related in some way",* while procedural knowledge is identified as *"is defined as the set of rules and algorithms used to solve math problems".*

As stated by Brown and Skow (2016), students' errors in mathematics are categorized into three types, which are:

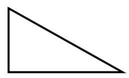
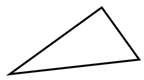
1. Factual mistakes are mistakes made by students as they are short of factual information, such as digit identification.
2. Procedural mistakes are errors caused by inaccuracy in applying mathematical procedure, such as decimal placement.
3. Conceptual mistakes occur when students have misconceptions or misunderstanding about the concepts related to the problem, such as the concept of how to add two fractions.

Cited from Brown and Skow's paper, the table below explains the examples of each type of mathematical mistake.

Table 1. Mathematical Errors and Its Examples

Factual Mistakes	Examples
No comprehension of number facts	3 + 5 = 4 2 – 7 = 5
Misidentifies signs	3 x 2 = 5
Misidentifies the value of digits	6 – 2 = 7 (students identify 6 as 9)
Counting errors	Miss a number or more in counting: 1, 2, 3, 5
Lack of mathematical terms	Students do not know the meaning of numerator and denominator
Lack of mathematical formulas	Students do not know the formula of triangle's area

Procedural Mistakes	**Examples**
Regrouping errors	24 Students forget the regroup <u>47 +</u> the tens 61
Performing incorrect operations	4 x 5 = 9 Students regard the multiplication as addition
Fraction errors	Students cannot find common denominator when adding and subtracting fractions $\frac{1}{3}+\frac{2}{5}=\frac{3}{8}$ Errors in multiplying fractions: $\frac{2}{4}x\frac{3}{4}=\frac{6}{4}$ Errors in dividing fractions: $\frac{6}{9}\div\frac{2}{3}=\frac{3}{3}$
Decimal errors	Students do not align decimal places when adding or subtracting 2.34 <u>47.1 +</u> 70.5

Conceptual Mistakes	**Examples**
Misconception of place value	12 <u>9 +</u> 102
Overgeneralization	In subtraction, student always put the greater number as the minuend. 10 – 24 = 14
Oversimplification	  Right triangle not a right triangle

Aim of the Study and Research Questions

The purpose of this study is to identify third grade students' errors in solving fractions problems involving comparison of fractions and addition of two fractions. The findings of this study will help teachers to deal with students' error and to improve the instructions in fractions. Thus, this study try to seek this questions What error types do students make in solving problems involving fractions?

Research Method

The purpose of this study was to investigate students' errors in tackling mathematical problems involving fractions. To achieve this goal, the researcher constructed a set of problems related to fractions. Then, the problems were validated by some expert supervisors and a teacher of the experiment class. This study was conducted in SD N Laboratorium UNESA Surabaya. The students participated in this study were students of Class 3A and 3B.

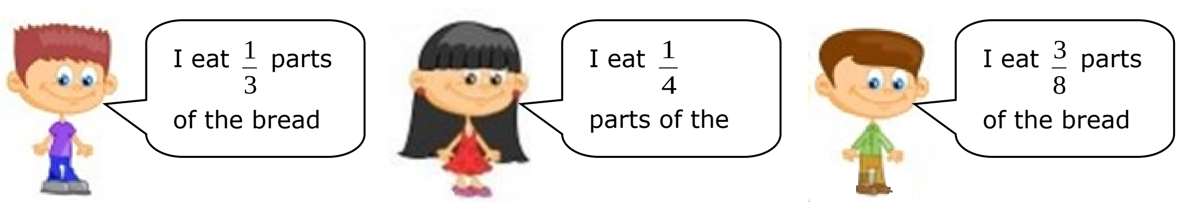
Results And Discussions

Comparison of Fractions

The problem relating comparison of fractions posed in this study was as follows:

Each Adi, Ara, and Fadil have a piece of bread.

Adi *Ara* *Fadil*



Order the children from the one who eats the smallest parts of bread to the one who eats the biggest parts of the bread!

Figure 2. The problem relating comparison of fractions

In this problem, students' solution were ranged. Most of them use cross-multiplication as below.

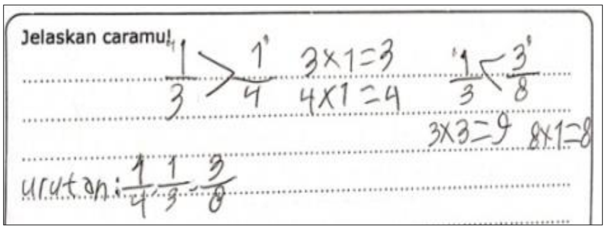


Figure 3. Students' strategy in comparing fractions

In comparing two fractions using that strategy, students multiplied the numerator of one fraction and the denominator of another fractions. The bigger multiplication result is, the bigger the value of the fractions.

However, some students employed some error strategies as following.

1. Subtracting the denominator from the numerator

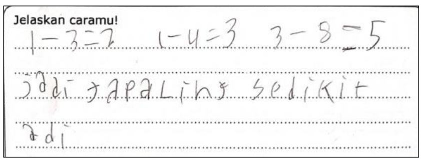


Figure 4. Students' error in comparing fractions (1)

In this approach, the student subtracted the numerator by the denominator of each fractions. It can inferred from the picture that students did both procedural and factual error. Firstly, he applied incorrect procedure to compare fractions. Secondly, when he subtract 3 from 1, he came to a wrong answer. He answered 2 instead of -2. In this case, he might not be aware of concept of number fact.

2. Using fractions representations to aid solving the problem

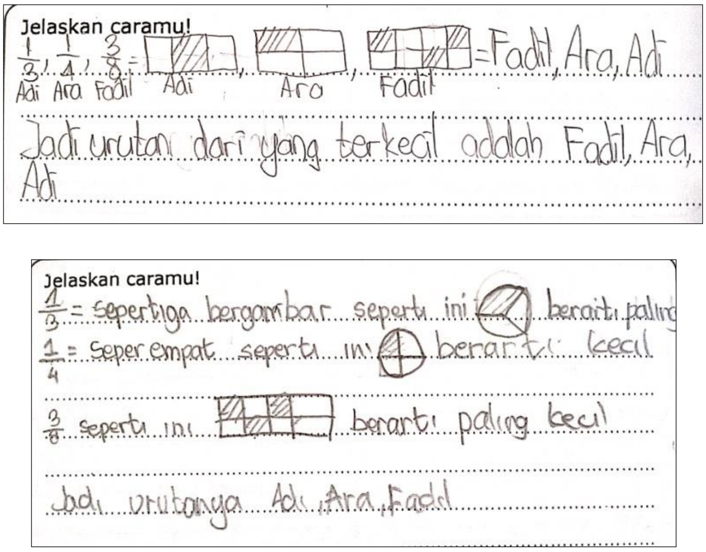


Figure 5. Students' error in comparing fractions (2)

It has been suggested by some studies that utilizing models or representations can help students understand mathematical concept, such as fractions (Cramer et al., 2008; van Galen et al., 2008). However, incorrect concept of how to use the models can be a hindrance in solving problems.

In the picture above, the students tried to use bar and circular models to represent the fractions. Nevertheless, it seemed that they did not aware that the models they used should be in the same shape and size. In the first picture, he used the same model, which was bar model, but he drew them in different sizes. As the result, the student missinterpret which fractions are bigger than the other. Meanwhile, the student of the second picture utilized different kind of models so he could not notice correctly which fractions are bigger than the other.

Addition of Fractions

In this topic, the researcher provided some problems involving different fractions. Bar model was presented in some problems in case students were willing to use them in solving the problems. Some students performed correct answers with various strategies, such as using bar model and using the concept of equivalent fractions. However, the other students carried out incorrect answers due to a lack of procedural and conceptual knowledge.

1. 'Top+top and bottom+bottom' strategy

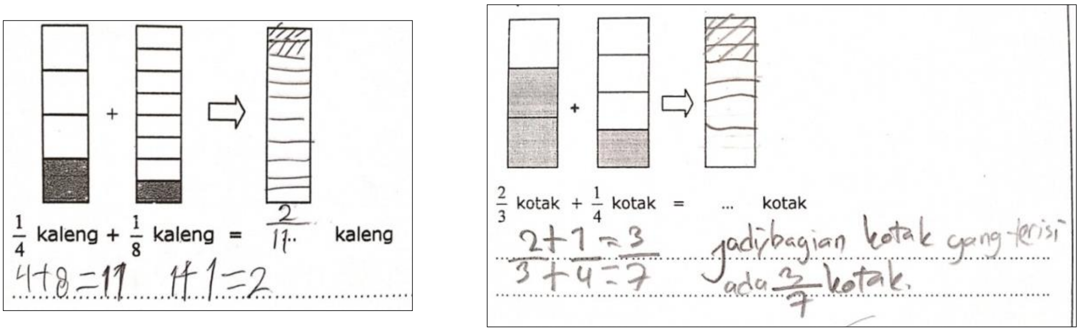


Figure 6. Students' error in adding fractions (1)

Simply adding the tops (numerators) together and the bottoms (denominators) together is the most common mistake by students in solving addition of fractions (Howard, 1991; Young-Loveridge, 2007). In the examples shown above, the students obtain the result of fractions addition problem by employing that strategy. Thereafter, they transferred the answer in the bar model available. Eventhough the result illustrated in the bar model is clearly confirmed that it was not reasonable, the students kept on their answer as they were lack of conceptual knowledge about the reasoning in adding fractions. For instance, when adding $\frac{2}{3}$ by $\frac{1}{4}$, the answer must be more than a half since $\frac{2}{3}$ itself is more than a half. Moreover, it seemed that they apply this strategy since they refer to addition of non-fractional numbers. Thus, they operated this procedural error.

2. Crossed-Multiplication Strategy

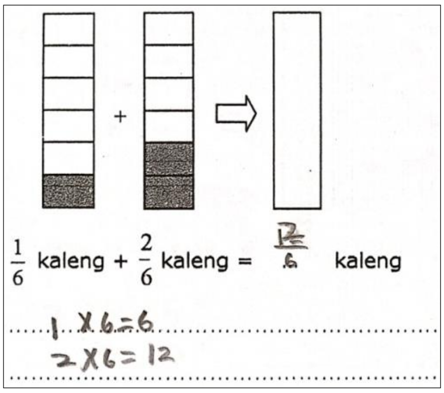


Figure 7. Students' error in adding fractions (2)

As can be seen in the picture, the students multipied the numerator of one fraction by the denominator of another fraction. Then, they put one of the multiplication result as the numerator and another as the denominator.

This procedural errors was caused by inaccurate generalization of a procedure used in comparing two fractions. Most of the students in this study were familiar with cross-multiplication strategy to compare two fractions as they teacher told them that strategy. Hence, some students generalized that this strategy holds in addition of fractions as well because they might not have any ideas of how to add fractions. Furthermore, there was a conceptual error as students cannot justify whether their answer was reasonable or not if it was represented in the bar model. Logically, the result of $\frac{1}{6}+\frac{2}{6}$ cannot be equal to two as both, $\frac{1}{6}$ and $\frac{2}{6}$, are less than one.

3. Crossed-addition strategy

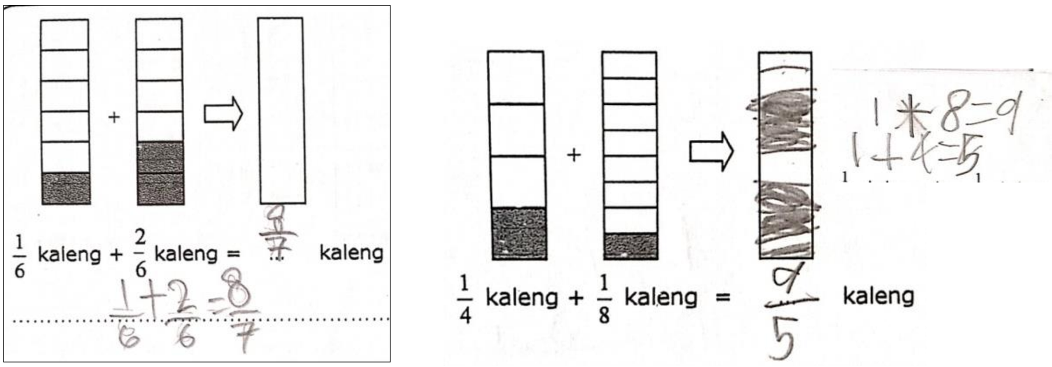


Figure 8. Students' error in adding fractions (3)

In this strategy, firstly they added the numerator of one fraction by the denominator of another fraction. Afterward, they put the result as the numerator while the other was put as the denominator. Similar with the previous type of error, the students tried to immitate the strategy they used in comparing fractions. However, as they were aware that the problems were about addition, they adapted the strategy by adjusting the operation from crossed-multiplication to crossed-addition.

4. Incorrect utilisation of bar model

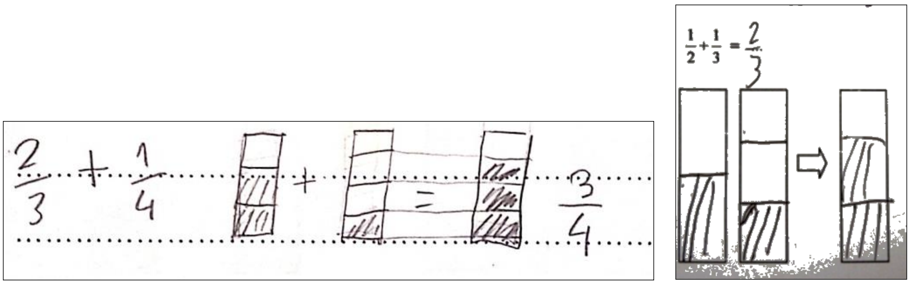


Figure 9. Students' error in adding fractions (4)

The students attemped to use bar model to figure out the problem but they failed. This mistake was categorized as procedural and conceptual mistake as the students were not aware of the importance of dividing all bars into the same equal parts. The answers they came up with were more reasonable than the previous mistakes. However, they were less accurate since the students did not partitioned the bars into equal parts.

Conclusion

Most mistakes employed by students in this study are procedural and conceptual mistakes. Some students did not know how to compare and add fractions; thus they applied any procedures they were familiar with. Those mistakes included inaccurate use of models, 'top+top and bottom+bottom' strategy, crossed-multiplication, and crossed-addition strategy. Some studies suggested that mathematical errors occured as teachers focused more on formal algorithm than on understanding underlying reasoning behind the concept (Lamon, 2001; Idris and Narayanan, 2011). Thus, in the future, teachers should put more emphasize on students' understanding and reasoning to avoid such mistakes.

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