

# THE EFFECT OF CRITICAL TRACTION IN COHESIVE ZONE MODEL FOR FATIGUE CRACK GROWTH RETARDATION

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## ABSTRACT

*A cohesive zone model for simulation of fatigue crack growth is presented. The cohesive zone model is one of many alternative approaches used to simulate fatigue crack growth. The model incorporates a relationship between cohesive traction and separation in the zone ahead of a crack tip. The model introduces irreversibility into the constitutive relationships by means of damage accumulation with cyclic loading. The traction-separation relationship underpinning the cohesive zone model is not required to follow a predetermined path, but is dependent on irreversibility introduced by decreasing a critical cohesive traction parameter. The approach can simulate fatigue crack growth without the need for re-meshing and caters for single overloading. This study shows the retardation phenomenon occurring in elastic plastic-materials due to single overloading. Increasing the value of critical cohesive traction increases the extent of plastic zone at the crack tip which causes the fatigue crack growth to retard. Plastic materials can generate a significant plastic zone at the crack which is shown to be well captured by the cohesive zone model approach.*

**Keywords:** *cohesive zone model, fatigue crack growth.*

## INTRODUCTION

The cohesive zone model is delivered to provide a realistic representation fracture mechanism and also delivers simplicity in simulations of complex cracking processes. From the continuum mechanics point of view, crack extension improve complex failure processes at the microscopic levels such as void nucleation, growth and coalescence in ductile metals, microcrack in ceramics and crazing in certain polymers. If the failure process is limited to a narrow band, such as crazing in polymers and necking in ductile thin-sheet materials as shown in Figure 1, the cohesive zone may be utilized to perform the narrow deformation band. The cohesive zone may be regarded as an approximate representation of the crack tip failure process zone, such as microcrack in brittle materials and void growth and coalescence in ductile metals as shown in Figure 2.

In general, cohesive zone approach assumes that ahead of the physical crack tip exists a cohesive zone, which consists of upper and lower surfaces called cohesive surfaces held by the cohesive traction. The cohesive traction is related to the separation displacement between the cohesive surfaces by a cohesive law. The application of external loads to the cracked body may cause the two cohesive surfaces to separate gradually and lead to crack growth when the separation of both surfaces at the tail of the cohesive zone (physical crack tip) reaches a critical value. Figure 3 shows a cohesive zone ahead of a crack where is the

cohesive traction and is the separation displacement of the cohesive surfaces.

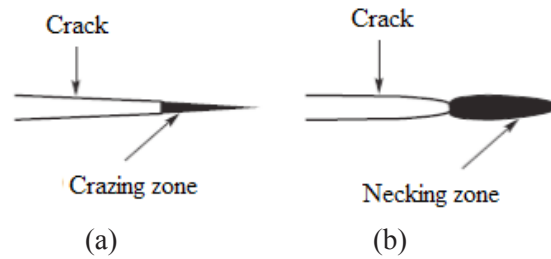


Figure 1 (a) Craze zone ahead of a crack in a polymer, (b) necking zone in a ductile thin-sheet material

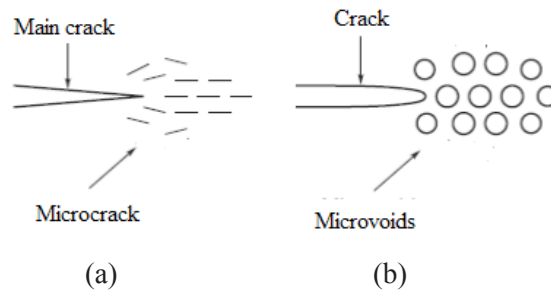


Figure 2 (a) Microcrack zone ahead of a crack in a brittle solid (b) voids in a ductile metal

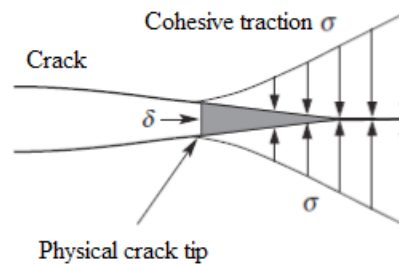


Figure.3 A cohesive zone ahead of a crack tip

The basic concept of cohesive zone model was introduced by Dugdale (1960) [1] and Barenblatt (1962) [2]. In these models, the crack is divided into two parts, one part of the crack surface is stress free, and the other part is loaded by cohesive stresses. Dugdale (1960) [1] introduced the finite stress to be the yield stress  $\sigma_y$  applicable only for plane stress, but in practice the crack-opening stresses can be much higher than by yield stress. Barenblatt (1962) [2] investigated the fracture of brittle materials, he made several assumptions about the cohesive stresses, i.e. the extension of the cohesive zone is constant for a given material and small compared to other dimensions, the stress in the cohesive zone follow a prescribed distribution yield stress,  $\sigma_y(x)$ , where  $x$  is ligament coordinate, which is specific for a given material but independent of the global loading conditions.

The first application of the cohesive stress at the crack tip in the finite element method and developed crack tip model that involves crack growth was introduced by Hillerborg (1976) [3]. He proposed a model which in some respects is similar to the Barenblatt's model for brittle material. The crack is assumed to extend when the stress at the crack tip reaches the tensile strength. When the crack surface opens the stress is not assumed to fall down to zero suddenly, but it will decrease with increasing separation. The stress reaches zero when the crack tip surface reaches prescribed separation distance.

In the current cohesive zone model, material separation and damage of the structure is classically described by interface elements. The interface elements open when damage occurs and completely lose their stiffness at failure so that continuum elements became disconnected. The crack is limited to propagate only along the element boundaries. The constitutive behaviour of a cohesive element is characterized by the relationship between a crack tip opening value or critical separation  $\delta_c$  and the critical cohesive traction  $\sigma_c$ .

A large number of phenomenological traction-separation laws have been proposed and illustrated in Figure 5.6 such as bilinear [4], trapezoidal [5], polynomial and exponential [6] as shown in Figure 4. The qualitative shape of these functions is similar. Starting at the traction free undeformed state, the value of cohesive traction increases with the separation of crack surfaces up to a maximum value  $\sigma_c$  and decreases to zero when complete separation occurs at specific critical separation value  $\delta_c$ . However, the effect of the cohesive law shape on numerical results of fracture simulations has been investigated by Nguyen (2001) [7]. He revealed that the shape of traction-separation curve does not significantly affect the separation materials behaviour in numerical results.

For fatigue analysis, irreversible cohesive zone model involves a damage evolution mechanism representing gradual degradation of the cohesive traction under cyclic loading. de-Andres (1999) [8] considered to add unloading conditions to a traction–separation law used in monotonic loading in conjunction with a cycle dependent damage variable to study fatigue crack growth under large scale yielding. Nguyen (2001) [7] developed a cohesive zone model using irreversible unload–load relations. The model was utilized to study fatigue crack growth in macroscopic aluminium specimens for constant amplitude loading and also to investigate the effects of overloads on fatigue crack growth. Roe and Siegmund (2003) [9] and Siegmund (2004) [10] proposed irreversible cohesive zone model involving a cyclic damage evolution rule for the cohesive strength to simulate fatigue crack growth.

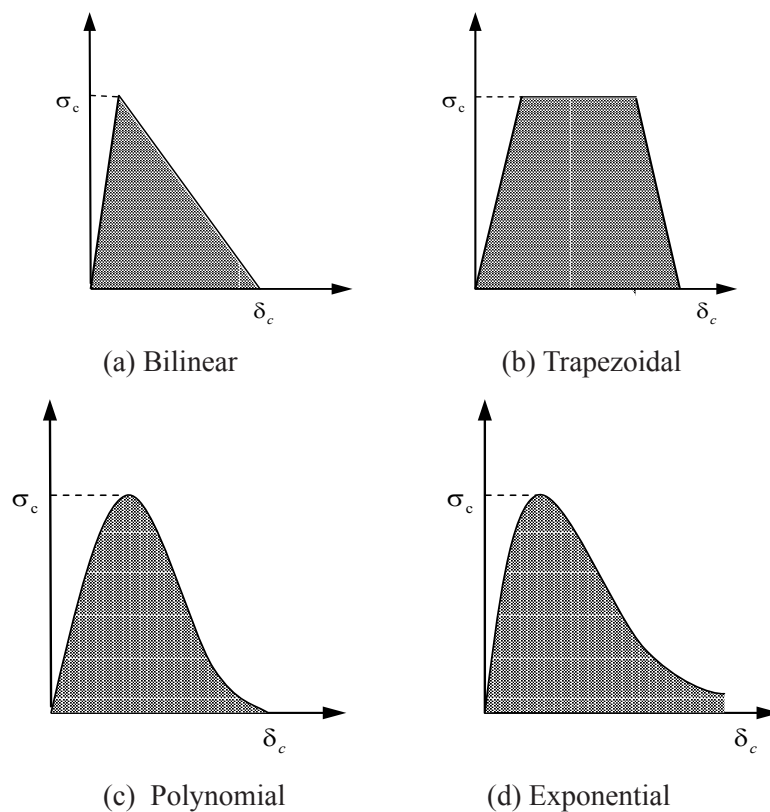


Figure.4 The various shapes of traction-separation law

Most irreversible cohesive zone models considered the separation will back to zero along with unloading condition. However, in the present study, it is assumed that normal displacement jump or separation under unloading condition is constant. This is caused by the normal separation undergo the plastic deformation in that condition. The irreversible cohesive zone model is defined as the accumulation of the normal displacement jump or normal separation under cyclic loading. The normal separation also relate to the degrading of cohesive traction.

Single overloading is typically characterised by one high peak load and then followed by the lower constant amplitude loading. The overloading may lead to retardation or acceleration of fatigue crack growth. This phenomenon is a significant unexplained aspect in fatigue crack growth [11]. Most approaches proposed and developed are empirical which account for the retardation phenomena by means of a plastic zone created due to the high peak loading [12].

In cohesive zone model, three parameters are involved in the evaluating of fatigue crack growth i.e. critical cohesive traction  $\sigma_c$ , cohesive energy or toughness and critical separation ( $\delta_c$ ). These parameters relate to the materials behavior in damage zone ahead of the crack tip. Since toughness is a well-established material property, the critical cohesive traction or normal separation is parameters which need to be specified. However, the critical cohesive traction  $\sigma_c$  is a parameter which dictates to some extent the plastic behavior at damage zone ahead of the crack tip. Its value is required to be higher than the yield stress  $\sigma_y$  for yielding to occur at the crack tip. The plastic zone at the crack tip is recognized to be one of the most important quantities regarding for the retardation and induce crack closure [13]. It is of interest therefore to consider the effect of various critical cohesive tractions  $\sigma_c$  on the fatigue crack growth retardation.

## THE OBJECTIVE

In this paper, the cohesive zone model in Code\_Aster is presented. The effect of critical cohesive traction on the fatigue crack growth rate by using the cohesive zone model due to the single overloading is analyzed.

## COHESIVE ELEMENT LAW

In this paper, the cohesive element law was developed by Laverne (2012) [14]. It is considered reasonable to assume that the opening crack dissipates energy proportional to its length. The dissipated energy is called the surface energy and is denoted as  $\Psi$ . The surface energy defined at a discontinuity  $\Gamma$  representing a crack is considered dependent on the normal displacement jump  $\delta_n$  of the crack surfaces and an internal variable  $\kappa$  which manages the crack irreversibility. The crack irreversibility is defined to be the accumulation of normal displacement jump during loading cycles.

In incremental terms the crack irreversibility is expressed as

$$\kappa^i = \kappa^{i-1} + \langle \delta_n^i - \delta_n^{i-1} \rangle \quad (1)$$

Where  $\langle \cdot \rangle$  is positive when its content are positive but zero otherwise. The surface energy is defined as follows

$$\Psi(\delta_n, \kappa) = \int_{\Gamma} \Pi(\delta_n, \kappa) d\Gamma \quad (2)$$

where the surface energy density  $\Pi$  is considered of the form

$$\Pi(\delta_n, \kappa) = \psi(\kappa) + I_{R^+}(\delta_n) \quad (3)$$

where  $\psi$  is a cohesive energy density and  $I_{R^+}$  is indicator function reflecting the impenetrability condition the crack surface.  $I_{R^+}$  is defined to be

$$\begin{aligned} I_{R^+}(\delta_n) &= \infty & \text{for } \delta_n < 0 \\ &= 0 & \text{for } \delta_n > 0 \end{aligned} \quad (4)$$

where the subscript n denotes the normal component ( $\delta_n = \underline{\delta} \cdot \underline{n}$  is the opening displacement or separation,  $\underline{n}$  is an unit normal at  $\Gamma$ ). The indicator function precludes inter penetration since negative displacement jumps would result in an infinite energy (hence not a minimum).

The cohesive energy density  $\psi$  can further be defined as

$$\begin{aligned} \psi(\kappa) &= \frac{-\sigma_c^2}{4G_c} \kappa^2 + \sigma_c \kappa & \text{for } \kappa \leq \frac{2G_c}{\sigma_c} \\ &= G_c & \text{for } \kappa \geq \frac{2G_c}{\sigma_c} \end{aligned} \quad (5)$$

where  $\sigma_c$  is the maximum traction or critical cohesive traction and  $G_c$  is the critical strain energy release rate also known as toughness or cohesive energy. The stress vector noted traction  $\underline{\tau}$  is defined as belonging to a differential of the surface energy density

$$\underline{\tau} = \partial \Pi(\delta, \kappa) \quad (6)$$

It is possible to define the normal cohesive traction to

$$\sigma_n(\delta_n) = \frac{\partial \psi}{\partial \delta_n} = \frac{\partial \psi}{\partial \kappa} \frac{\partial \kappa}{\partial \delta_n} \quad (7)$$

Since differentiation of Equation (7) reveals

$$\begin{aligned} \frac{\partial \psi}{\partial \kappa} &= \sigma_c \left( 1 - \frac{\kappa}{\delta_c} \right) & \text{for } \kappa < \delta_c \\ &= 0 & \text{for } \kappa \geq \delta_c \\ \frac{\partial \kappa}{\partial \delta_n} &= 1 & \text{for } \delta_n > 0 \text{ and } \delta_c \neq 0 \\ &= 0 & \text{for } \delta_n < 0 \text{ and } \delta_c \neq 0 \end{aligned}$$

where  $\delta_c = 2G_c \cdot \sigma_c^{-1}$  is the critical separation that means beyond this value the rupture of materials occurred.  $\delta_n$  is the rate of normal separation. The cohesive law can be conveniently classified into three stages: initial stage, damage and post fracture as illustrated in Figure 5.

In initial stage, there is no cumulative irreversibility so  $\kappa^{i-1} = 0$ , and consequently the normal cohesive traction is function of current normal separation which can be defined as follows

$$\begin{aligned} \sigma_n(\delta_n^i) &= \sigma_c \left( 1 - \frac{\delta_n^i}{\delta_c} \right) && \text{for } 0 < \delta_n^i \leq \delta_c \\ &= 0 && \text{for } \delta_n^i \geq \delta_c \end{aligned} \quad (8)$$

and initiation period where  $\sigma_n(0) \leq \sigma_c$ . The damage stage involves irreversibility, so  $0 < \kappa^{i-1} < \delta_c$ , and normal separation is defined as

$$\begin{aligned} \sigma_n(\delta_n^i) &= 0 && \text{for } 0 < \delta_n^i \leq \delta_n^{i-1} \\ &= \sigma_c \left( 1 - \frac{\kappa^{i-1} + \delta_n^i - \delta_n^{i-1}}{\delta_c} \right) && \text{for } \delta_n^{i-1} < \delta_n^i < d_c \\ &= 0 && \text{for } \delta_n^i > d_c \end{aligned} \quad (9)$$

where  $d_c = (\delta_c S_c) \cdot \sigma_c^{-1}$  and  $S_c = \sigma_c \cdot \left\{ 1 - \left( (\kappa^{i-1} - \delta_n^{i-1}) \cdot \delta_c^{-1} \right) \right\}$ .

In the post fracture stage identified by  $\kappa^{i-1} \geq \delta_c$ , where the normal separation can be defined as

$$\sigma_n(\delta_n^i) = 0 \quad \text{for } 0 < \delta_n^i \quad (10)$$

Cycle is presented in Figure 5.8 where the  $0 \leq \sigma_n(\delta_n^i) < \sigma_c \left( 1 - (\kappa^{i-1}) \cdot \delta_c^{-1} \right)$  applied. Figure 6 presents cyclic condition in the traction-separation curve. One first cycle is identified via path 0-1-2-3-0 then second cycle followed by 0-3-2-4-3 and so on. Since the area under one looping is defined as energy dissipated or separation energy, a crack begins propagation when the accumulation of separation energy every cycle reaches the cohesive energy  $G_c$ . The separation energy dissipated for the one cycle loading can be calculated by

$$\Delta G = \sigma_c \left( 1 - \frac{\sigma_c}{4G_c} \right) \quad (11)$$

and then the normalized separation energy is equal to

$$\frac{\Delta G}{G_c} = \frac{\sigma_c}{G_c} \left( 1 - \frac{\sigma_c}{4G_c} \right) \quad (12)$$

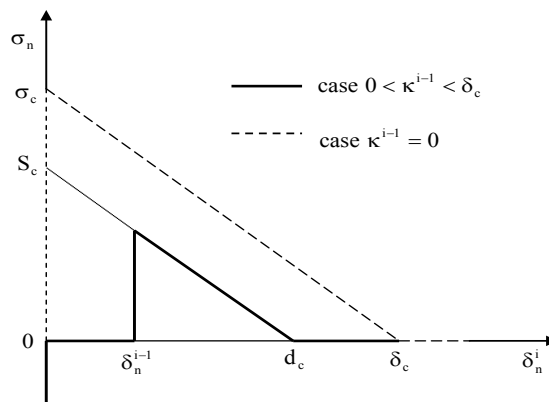


Figure.5 Schematic of cohesive law

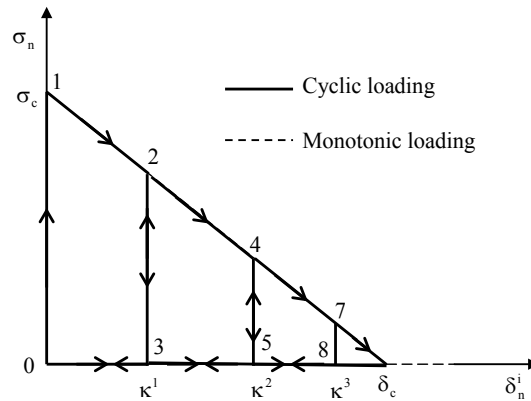


Figure 6. Evolution of traction during monotonic and cyclic loading with the cohesive-traction law

**MODEL DESCRIPTION**

In the present work, a model consisting of 17.7 mm crack length with 50 mm of height (h) and 52 mm of width (w) is analyzed. The crack propagation is assumed to proceed along the axis of symmetry of the model as shown in Figure 7 (a). The assumed symmetry means that only half the model is simulated. The cohesive interfaces of 163 elements are placed along the symmetry line from the initial crack tip to the outer boundary. The model is meshed with 12680 quadratic-triangular elements as shown in Figure 7 (b). The mechanical properties of the material used for linear-elastic analysis are:  $E = 72 \text{ GPa}$ ,  $\sigma_y = 300 \text{ MPa}$ ,  $\nu = 0.3$  and  $G_c = 9.5 \text{ N/mm}$

A single stress overloading ( $S_{ol}$ ) of 26.25 MPa is applied at the first cycle and then followed by a stress loading ( $S_{max}$ ) of 15.0 MPa applied uniformly on the top edge of the plate as a sinusoidal function as shown on Figure 7 (c). An overload ratio  $SOL = S_{ol} / S_{max} = 1.5$  with a load ratio  $R = S_{min} / S_{max} = 0$  is applied. In this test case, the various critical traction (CS) used to examine the effect on crack propagation are 310 MPa, 320 MPa, 330 MPa, 340 MPa and 350 MPa, respectively.

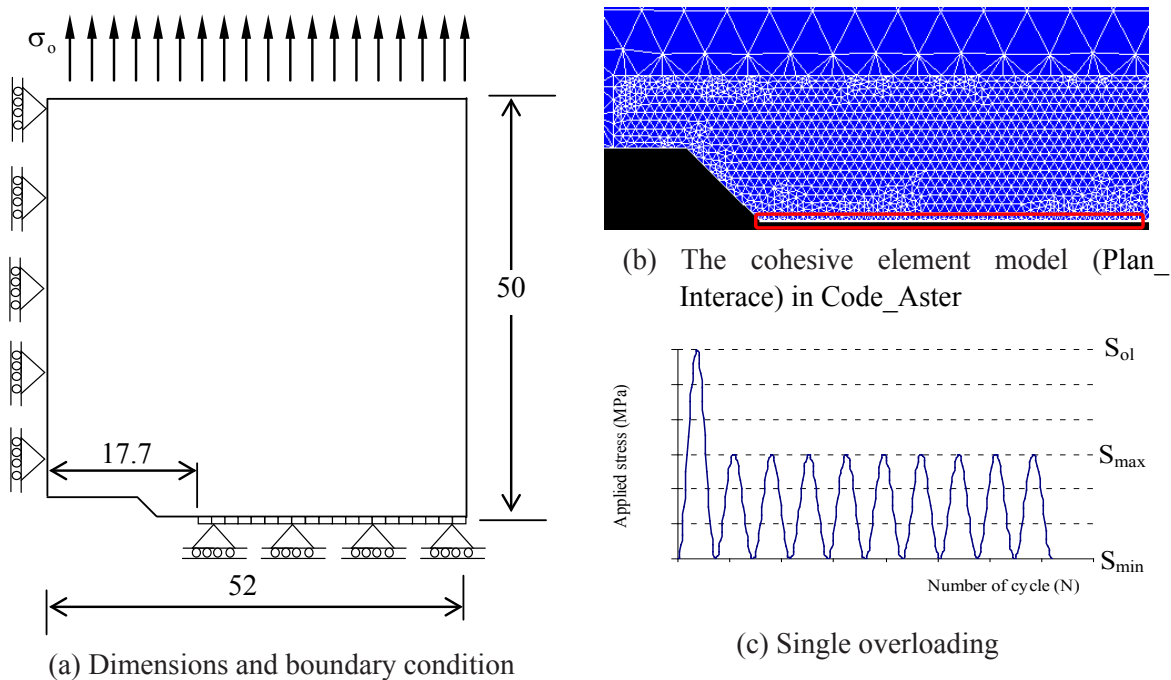


Figure 7. The schematics of models

## RESULTS AND DISCUSSION

The crack length with respect to the number of cycle in various critical cohesive tractions  $\sigma_c$  is depicted in from Figure 8 to 12. The figures show the comparison of fatigue crack growth without overloading (SOL-1) and with single overloading (SOL-1.75). It can be seen from these figures that for the critical cohesive traction CS-310 and CS-320 MPa, there is no retardation for an applied single overload as shown in Figure 8 and 9, respectively. The phenomenon of retardation appears for the critical cohesive traction of 330 MPa and 340 MPa, and is particularly occurs when the critical stress value is 350 MPa as shown in Figure 10, 11 and 12, respectively.

For the case study pertaining for various critical cohesive tractions  $\sigma_c$ , it has been found that generally the critical cohesive traction affects the fatigue crack growth retardation. If the value of critical traction is close to the yield stress, it will not retard fatigue crack growth. Increasing the value of critical cohesive traction increases the extent of plastic zone at the crack tip. The plastic zone at the crack tip is recognized to be one of the most important quantities regarding for the crack growth retardation.

## CONCLUSION

One of the more significant findings to emerge from this case study is that the cohesive zone model in Code\_Aster can simulate the effect of a single overloading to retard the crack growth rate.

The critical cohesive traction affects the fatigue crack growth retardation. Increasing the value of critical cohesive traction increases the extent of plastic zone at the crack tip. which causes the fatigue crack growth retardation.

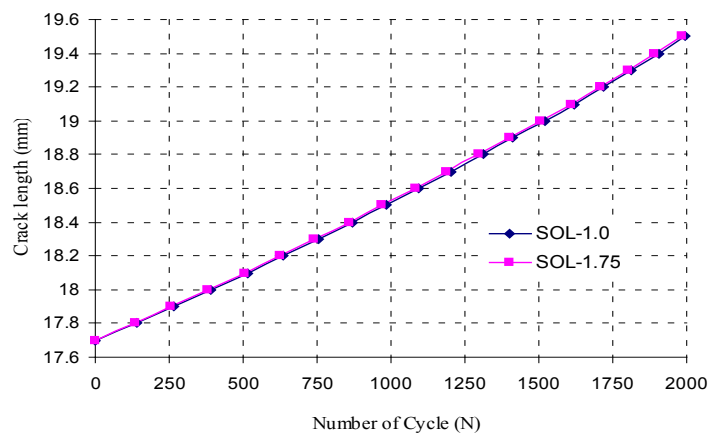


Figure 8. The comparison of the crack growth rate for difference type of loading at critical cohesive traction (CS) of 310 MPa



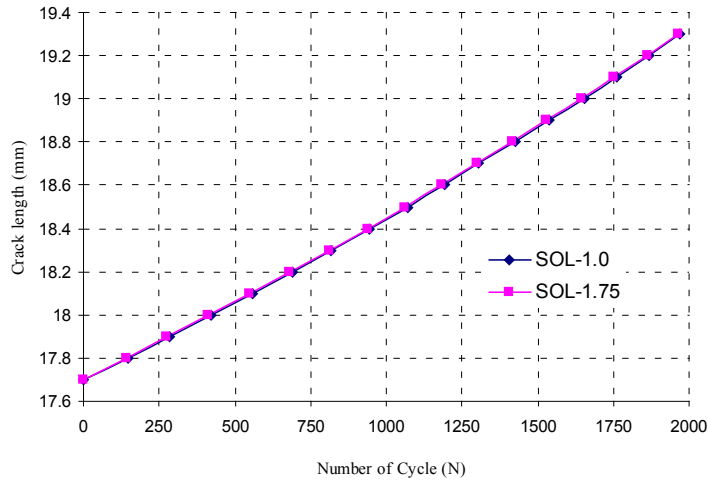


Figure 9. The comparison of the crack growth rate for difference type of loading at critical stress (CS) of 320 MPa

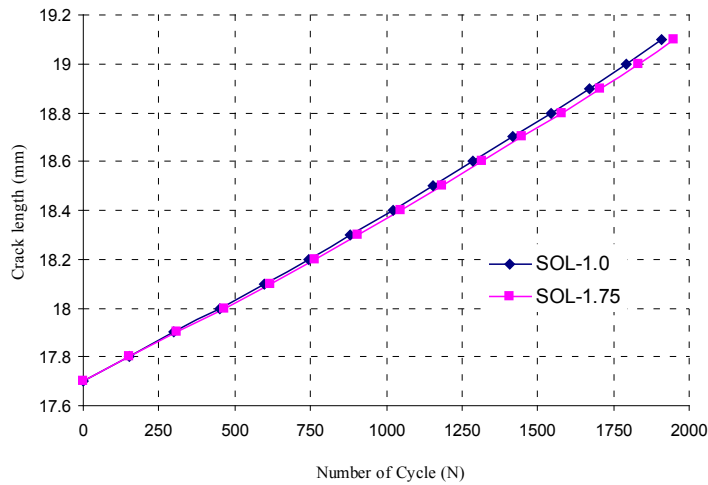


Figure 10. The comparison of the crack growth rate for difference type of loading at critical cohesive traction (CS) of 330 MPa

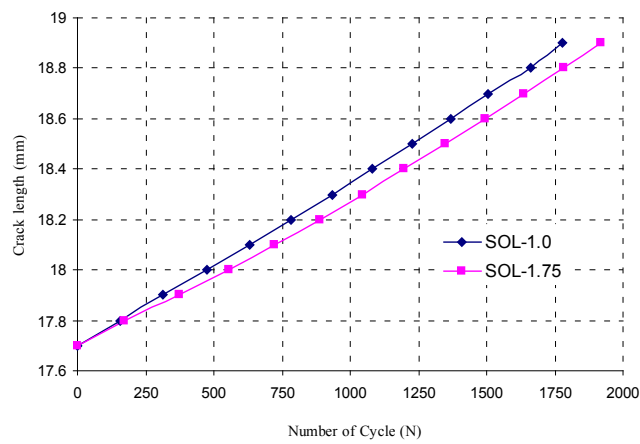


Figure 11. The comparison of the crack growth rate for difference type of loading at critical cohesive traction (CS) of 340 MPa

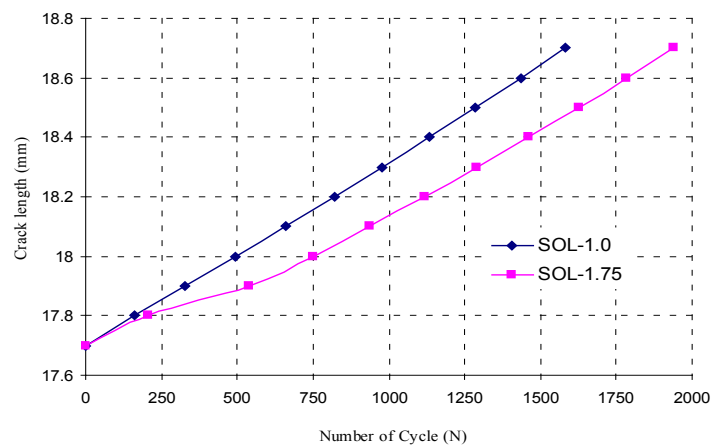


Figure 12. The comparison of the crack growth rate for difference type of loading at critical cohesive traction (CS) of 350 MPa

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