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A Specific Kind of Representation: How Systematics May Ease Cognitive Overload

Özlem Cezikturk
Marmara University, Turkey
Corresponding author's email: ozlem.cezikturk@marmara.edu.tr

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ABSTRACT

Multiple representations are beneficial for meaningful understanding. However, three or more representations may add to the cognitive overload of students, if not in interactive diagrams and dynamic geometry. How a well-known representation consisting of more than 3 or more representational registers may overcome the problem of cognitive overload without being too complicated. In this study, an old but well-structured representation that was used even over 40 years was analyzed. The critical points of a function, asymptotes, x / y intercepts, inflection points, and graphing can be identified easily. It is prepared in the form of a table and the factors of the first derivative of the function and the second derivative and their roots indicate the function's increasing and decreasing intervals and its graph. This representation is very systematic and it acts like a method to draw the function's graph with no fault possible. Yet, besides being used for many years, is still used for courses like Calculus etc. We argue that cognitive overload theory cannot alter this representation due to its systematic nature. As content analysis, some examples of this representation are shared via the reader, and some qualitative aspects about it are analyzed. Finally, its systematicity, well-structured nature, and nature in reducing extraneous cognitive load are emphasized. Important thing here is that, it is very strategic to not to lose some representations for the sake of new ones if their value is already known but not discussed too much.

***Keywords**: Multiple Representations, Cognitive Overload Theory, Systemic Thinking*

INTRODUCTION

A representation is a way of demonstrating mathematics with a meaning building aim. In a way, a representation may be thought of as a system. If a representation carries too much information without necessary relations, it is hard for students to make required meaning out of it. If it carries too less information with no necessary background for concept building, another misconception may occur. Hence, it is like a threshold for students for meaning-making. And it is a fact that there is no mathematical object that is free from its representation.

Systems thinking in mathematics education is a way of thinking that flourished with structuralism though the method to obtain is different. When investigated, systems theory and thinking deal with the operating system while structuralism deals with the inner abstract model. Juxtaposing the elements of the system, to understand its togetherness better and how it works and to analyze the inner model if it is a structure, connects two stances (Wieting, 1976). Systems thinking is thought as one of the highest levels of mathematical thinking. Hence it is challenging to obtain and carry. In the last phase of the Van Hiele Geometric Thinking Test, last five questions are for systematic thinking. Unfortunately, in most of the research carried, this phase seems to be something that all students never achieved. In research we carried out many years ago, we were anxious about systems thinking and its effect on Wasan geometry problems (Çeziktürk-Kipel, & Özdemir, 2016). The result was not optimistic. Not so many students could come to that phase as van Hiele suggested once (van Hiele, 1986).

Another perspective may be the need for enough abstraction in the representations. By proper representation, we mostly try to decontextualize concepts so that students would use them in any possible place without hesitation (Lave, 1988, cited in Mitchelmore & White, 2007). Noss & Hayles (1996, cited in Mitchelmore, & White, 2007) add that as in the presence of a structure, the learner can draw up and reconstruct as a help. Mitchelmore & White (2007) propose that teachers familiarize the students with the same structure in various contexts and make them recognize the whole concept by itself. The same can be considered as recognizing the possible structure. Davydov (1990, cited in Mitchelmore & White, 2007) notes that systemic analysis of relations between objects produces theoretical (scientific) concepts. Vygotsky (cited in Mitchlemore & White, 2007) similarly differentiated between every day / scientific concepts, and he thought that a system of relations among concepts produces scientific ones.

Individual students, teachers, and classrooms can be thought of as complex systems. Substantial learning environments include higher-order thinking, mathematicizing, exploring, communicating, and reasoning. Hence, basic skills and procedural skeletons must be combined in such learning environments (Wittman, 2021).

Pino- Fan, Guzman, Duval & Font (2015) stressed the semiotic representations for students to understand any concept possible. They concluded that a register of semiotic representations and an onto-semiotic approach should be combined for a full-fledged representational system. As Duval (1995, cited in Pino et al., 2015) argues, no knowledge is differentiated from a representational activity. Specificities of a semiotic representation include different registers separately working for different aims but explicitly doing their stuff, some not being able to carry some parts of the system, and the object itself. They argue that a trace is needed to see the cognitive activities related to representations. Moreover, altogether new objects emerge from the operative and discursive math practices systems by structures. A semiotic systematizes registers' analysis, and it is a need.

İncikabı & Biber (2018) &similarly Çeziktürk, Aras, Zengin, Ayrancıoğlu, & Aslan (2019) stress that some representations are used more than the others in textbooks as in algebraic and verbal representations. They state that tabloid, graphical, and real-life representations are rarely seen in textbooks and hence need to be increased by number. They also posit that treatments and conversions may be a good solution for increasing systematizing in the representations' structures.

There are some numerical restrictions on using the representational registers, such as Paivio's Dual Coding Theory and Sweller's theory of cognitive overload. Paivio (2006) stated in his research that two representational registers simultaneously could be a solution. He was suggesting images and verbal system. However, he never neglected the computation in the representational registers. He believed that concrete objects are easier to learn and imagery enables this concretization than abstraction. Sweller (1988, 1994, 1998), in his theory of Cognitive Overload, stated that three or more representations simultaneously could produce cognitive overload for the students while processing.

Jong (2010) sees function as going from one system to another. He mentions this transition as structural and operational. He states that cognitive capacity in working memory is limited and too much capacity requirement causes problems in learning. As a remedy, he suggests we need instructional systems that optimize the use of working memory capacity and avoid cognitive overload. He mentions three types of cognitive load intrinsic, extraneous, and germane cognitive load. For intrinsic he indicates inherent characteristics of content. For extraneous instructional material and for germane learning, learning processes are thought for the focus of the cognitive load. He proposes sequencing the material from simple to complex, splitting the attention effect but in an integrated way, redundancy but meaningfully, means-end analysis, removing the affordances, and focusing on learning processes such as interpreting, exemplifying, classifying, inferring, differentiating, and organizing. Here, organizing refers to organizing selected words and images and integrating modular to a whole. Here, he stresses the careful checking of mental activities not interfering with the construction or automation of schemas. In other words, well designed instructional formats lead to processes adding up to schema construction. Gerjets & Scheiter (2003, cited in Jong, 2010) structure emphasizing rather than surface emphasizing is a choice instruction designer should make to. Jong (2010) also posits interactivity as a reducer of cognitive load by allowing learners to digest and integrate segment by segment the information before going to the next.

Here a question arises: How can we treat some old representations that we still use without hesitantly since they act systematically as possible. How do they reduce cognitive overload if there is a role in it? Our research problem arose from this question when we first encountered the specific representation that we analyzed in this study. As the problem of the study, a specific kind of representation from past and present has been identified, and it is analyzed for what makes this representation much less problematic for cognitive overload than other representations in terms of its systematicity.

METHOD

It is designed as a qualitative inquiry into a well-known representation structure by some specific example functions. It does not matter which functions are used in this explanation. What matters is how systematic is the structure of the representation thought. In this qualitative inquiry, the content analysis approach is used to analyze the structure of the representation X. We could not assign a name to it hence we named it representation X. It is also a matter of fact that there may be other representations X if we can found over the curriculum.

2.1. Data Collection Tools

Data collecting tools are mostly our old lycee math notebooks and much older course books of mathematics (Akçabay, 1961; Baykara, 1942; Bogomolov, 1986; Çanakçı 2006; Tanın 1962; Thomas, Weir, Hass, & Heil, 2014 and Sezer, 1986). Some new and well-known mathematics books were also used like “Thomas’Calculus” as well. Unfortunately, for data collection, the books at hand established the data to be studied. In other words, purposeful data collection was at hand.

2.2. Data collection Procedure

After reconciling with this specific representation X at hand, similar and different representations from old and new books were searched. It turned out that ancient books and old course notes consisted of this representation. Hence it was a well-known and used representation for decades. If it is working, it may be a good idea to keep that representation in the future mathematics curricula as well, even though we will see that some new books are searching for new ways to teach similar concepts and relations. Of course, one could find many different books, including and not including this type of representation X, but it is a fact that it carries some systematic way of thinking and this in turn possibly helps students /performers to function adequately within this topic of functions' graphing.

2.3. Data Analysis

In the analysis of the data, the content analysis approach was used. All those course books and notebooks were searched for similar representations from the beginning to the end page. The author is a graduate of a regular high school at that time, hence can argue that most typical high school graduates possibly used this type as their teacher has used it in the mathematics classroom.

For systematic thinking, Maani & Maharaj (2002) framework was used. The framework included dynamic, system as a cause, forest thinking, operational, closed-loop, quantitative, and scientific dimensions for systematic thinking in general. Actually, for their research aim, they neglected the last two dimensions, but we could include those two for our research purposes. If the author thinks of herself as one performer of this kind of representation, I think that would not be a misleading result since those notebooks are hers and those books were her parents' old math textbooks and may be considered timely data possible.

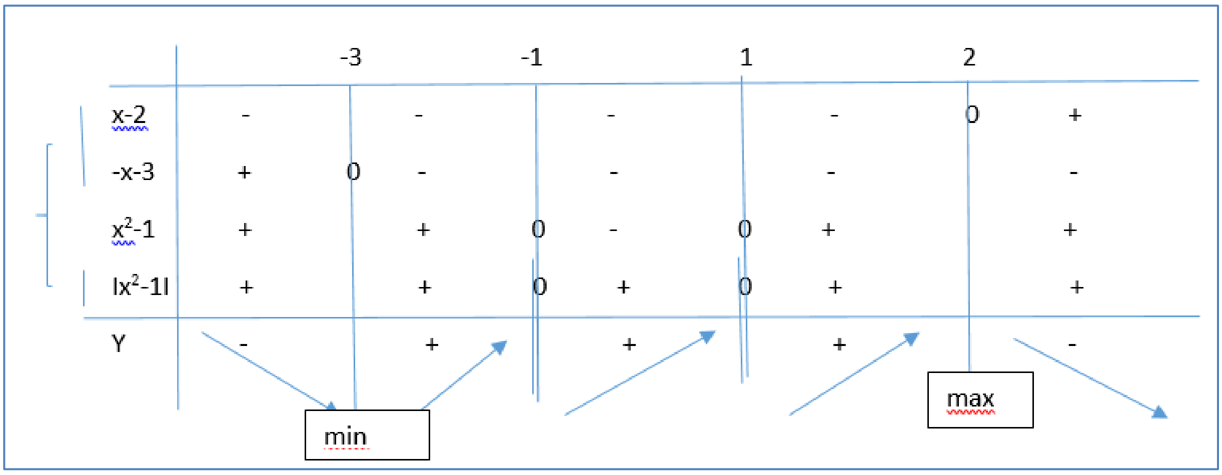


Figure 1. An example function with the so called representation

Meanwhile, a special kind of representation took our attention. They were discrete, very systematic, and full of thinking. We could be doing and carrying any kind of questions with them easily for so many years. We even ended up some of the old course books from the past (Akçabay, 1961; Baykara, 1942; Bogomolov, 1986; Çanakçı 2006; Tanın 1962; Thomas, Weir, Hass, & Heil, 2014) and they included this type of representation. My old course notebook, which is almost 35 years old (Sezer, 1986), had some examples of this type of representation in it. I graduated from a Normal Lycee in İstanbul. This year, I am teaching a calculus course and an analytic course, and I still use representation X, and many textbooks use this kind of representation without hesitation. So what makes this kind of representation so different from others? And what makes this representation a way of overcoming the problem of cognitive overload? I will try to support my classroom observations with related literature. For example, let us analyze this type of representation for those questions. Here, we can see a complex function of square root and absolute value and rational function in it. Hence, we need to consider many factors while graphing such a function. y= $\sqrt{\frac{(x-2)(-x-3)}{Ix^2-1I}}$. The representation X is in Figure 1, and the analysis of this representation is in the following paragraph.

The function above is a square root function; hence, it cannot take negative values inside. Those two parallel blue lines are for the function's domain due to the absolute value in the denominator. Arrows are for original functions' increasing and decreasing intervals, and min /max indicates the possible extremum points. We get not only critical points, signs of the parts of the function, the place of the function graphic on the Cartesian coordinate plane, a good analysis of different kinds of functions' sign problems (like absolute value and square root or even ln x), points of discontinuity, limits at critical points from left and right, roots of the function (where the function passes through x axis), but also points that makes denominator zero (indefiniteness problem), etc. There can be seen so much knowledge in one representation.

In the first raw (Figure 1), one can find the roots of the factor functions of the whole function. It is a complex function regarding the factors, absolute value, and the outside square root. From the second raw in (Figure 1), for four rows, factor functions' behaviors are analyzed concerning the roots of the functions. For example, 2 makes x-2 zero and does not cause any significant change for the other factors, but 3 is a root of –x-3 and makes a sign change for –x-3 from left to right. (x^2-1) has two roots: -1 and 1, and between these roots, the sign of the function is negative but otherwise positive. Absolute value make the factor (x^2-1) positive all the time. At the last raw, we may find the sign of the whole function. From this table we may even end up with a piecewise function representation. In addition, it is another benefit of this representation. Nevertheless, since the whole function is a square root function, its domain should not take negative values and this restricts oneself into [-3,2]< – >{-1,1} interval. This interval becomes the function's domain as we can detect from this representation. To get the range of the function it is enough to replace x with -3 and 2 and see the results: as 0. By intuition one could say that between -1 and 1 the function could get a top value or between -3 and 2, there could be another maximum. Hence, it may be good to check x=0 and x=-1/2 to see the possible max values for y there. For both these two values; the result is √6, hence we may say that the range of the function is [0, √6]. Those ± 1 makes the whole function indefinite, hence we may expect some asymptotes in those points. Or in other words, some jumping points (or discontinuity points) for the function y may exist.

$\lim_{x_{>}-1-} f(x) = +\infty$ and $\lim_{x\to-1+} f(x)= +\infty$, and $\lim_{x\to1-} f(x)= +\infty$ $\lim_{x\to1+} f(x)= +\infty$ could be stated. Even though we checked all four points as -3,-1,1,2 as critical points, our result is -1 and 1 are critical points and the others are only points of intersection with x axis. We may even wait for some kind of symmetry due to Ix^2-1I factor since the parabola it represents is reflected via x axis due to the absolute value (Figure 2).

One can see that so much information is given in a representation in figure 1, and it leaves no blank space for the learner to understand the concept of the specific function. The most important thing is that this representation is systematic and step by step, making it easier for the students to understand. We can understand this even from the workout of the elements of the representation X in conjunction with each other. Graphical representation in Figure 2 is a result of the representation of Figure 1, not inside of it. However, it helps students to imagine it easily mentally.

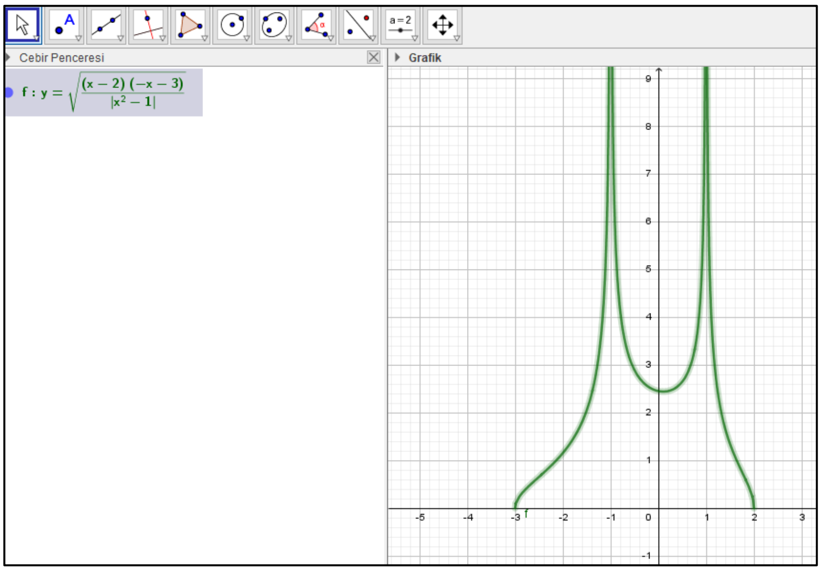


Figure 2. GeoGebra drawing of the example function

Understanding is harsh for functions' thinking and for mathematical topics where it needs more than two or three kinds of representation simultaneously. And especially when there is differentiation involved, the topic becomes difficult to process (Orhun, 2012). The goal of the study is to analyze a specific systematically driven representation X so that we would not lose the chance to see if it may be a solution to Sweller's (1988) well-cited problem of Cognitive Over Load. If some specific representations work without hesitation for students and even for multiple representations, we must detect them and make them well known for all mathematics education. However, we want our students to differentiate between being systematically driven and straightforward. Hence, it is our job to find what works and does not work from the curriculum and stress those over the others. In representation X, we see the algebraic register of representation in the + and -. We recognize the representation X as a table. And min and max identifications flourish as the graphical nature of the representation X. Besides these parts, the whole representation acts as a complex yet meaningful representation. It is very dynamic in the sense of the views of Maani & Maharaj (2002), there is cause and effect relationships, and complex as a forest yet acts as a whole by its significant elements.

RESULTS & DISCUSSION

3.1. Results

Cognition is something that we visualize with schema theory. In schema theory, new knowledge is supposed to be built upon the existence of the old knowledge if the structure is well established. Here, with this kind of representation, this requirement diminishes. Nobody asks for the prior knowledge status but what is asked is how this representation is built on top of that. It may be related to systematics.

Some researchers say that students do not feel comfortable with the derivative function, and it should not be used for graphing functions (Orhun, 2012). However, the derivative topic is a hustle when it is so differentiated from the curriculum. Hence, it may be good to tie it to the graphs of functions as in this representation.

With a very systematically organized representation like this, since everything is well organized and well structured, no fractures happen in the knowledge base. Everything is built on top of prior knowledge as it is supposed to and where it is supposed to.

In his theory of Cognitive Overload, Sweller (1988, 1994, 1998) stated that people have problems with three or more kinds of representations at the same time while processing. However,the authors believe he must have skipped this representation type X. Before his view, the Dual Coding Theory of Paivio (p.197 cited in Paivio, 2006) stated that two kinds of representations were good hand in hand while processing. In the above representation, it can be seen many representations at once. One can draw the function graph just by looking at this table. It may not be exact, but the graph would give everything needed inside.

So, what makes this representation X different than Sweller's and Paivio's views? Not difficult to see that its systematicity is a factor. As in Maani & Maharaj (2002), the systems thinking framework includes a dynamic, system as a cause, forest thinking, operational, closed-loop, quantitative, and scientific dimensions. In this example representation X, students use it in a timely and actively manner hence dynamic; different causes as interactive relations are given beforehand; students see the big picture not only the fragmented parts; there are parts where some operations are needed as sign multiplication; it asks for coming back if there is a problem with the drawing or if somethings do not end up, and it is scientific since each relation is scientifically bounded.

Furthermore, it is undoubtedly a semiotic representation, as Pino et al (2015) suggested. It stands for some specific concepts like critical points, multiplicative factors of function, roots of a function, indefinite points, maxima and minima, and some relations such as increasing and decreasing intervals, points that are making function zero, asymptotes and infinity, etc. This representation is surely disadvantaged when we think of what İncikabı & Biber (2018) and Çeziktürk et al. (2019) due to tables and graphics involved in it. It could carry a substantial learning environment in a course if the teacher uses it in the classwork (Wittman, 2021). It is partially abstract also, but it certainly does not carry a disadvantage since many applications are necessary. More than two registers are involved, at least algebraic, tabloid, graphical, and imagery. Nevertheless, it works by overwriting Paivio's (2006) and Sweller's (1999) ideas.

An excellent one-to-one example of this representation could be found in Canakcı's (2006) university exam test book: ÖSS MAT 5 (Figure 3). Here the derivative function is given first, and its roots and then the representation explains the signs of the derivative function in between the roots of the derivative. In these intervals, the actual function's increase and decrease directions are given; hence, extremum points are shared, and vice versa as maxima at x= $-2\sqrt{3}$ and minima at x= $2\sqrt{3}$. Maxima and minima points are stated as the related y's are given as much as the inflection points and asymptotes in the graph underneath. One could argue that separate paragraphs differentiate these two, but they are part of a great representation. Because all this information from the table makes it easier to draw the graph of the function, even though it is a complex function to be thought of firsthand. The function he was referring to was $f(x)=\frac{x^3}{x^2-4}$.

In the book of Akçabay (1961), there are three examples of these types of representations: one with inflection points (Figure 4), one with similar footage (Figure 5), and one with a whole explanation of the graph and important variables (Figure 6). In Figure 4, the actual function and its derivatives are given. Since the derivative function is always positive, only "+" is in the table. In addition, this demonstrates an increase for the actual function, always with an inflection point at x=0 in between. We understand that there is an inflection point at x=0 from the second derivative 6x being 0 at x=0. In figure 5, we see the same representation as of Çanakçı for $y= x^2-5x+4$. The only difference is that infinity boundaries show limits on the upmost left and limits on the upmost right points where the function diverges or converges. This time, the functions' derivative has only one root as x=5/2 for the minima, as shown in the graph on the right hand side of Figure 5. Here, we can also see the y-intercept as (0,4) and two roots of the function 1 and 4 as also x-intercepts. We can get these small variables from this type of representation quickly. İf the students, see the big picture (if the teacher stresses), the relations between critical points and the functions' properties are established and be clearly seen as in the figure.

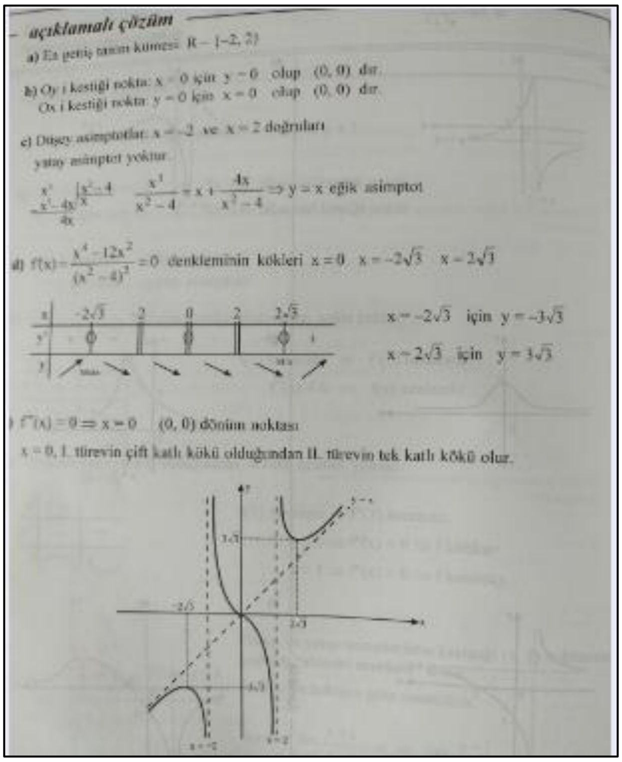


Figure 3. Çanakçı(2006) Example representation.p.264

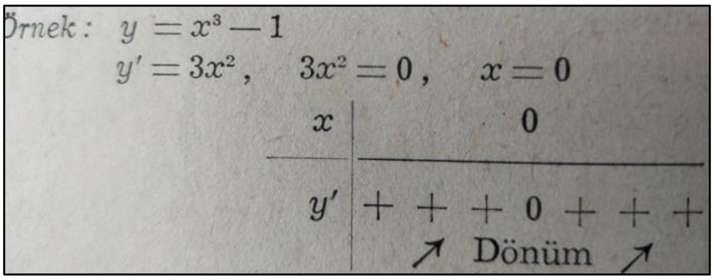


Figure 4. Akçabay (1961) same systematic for inflection points, p.130

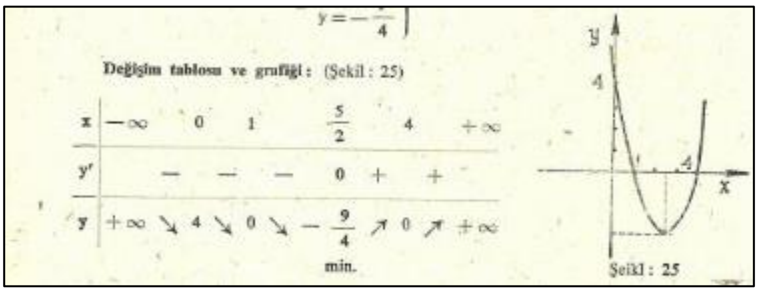


Figure 5. Akçabay (1961)y=x^2-5x+4, p.134

In the third representation from Akcabay (1961)(Figure 6), the function this time is a polynomial with the degree four and highly complex. It would be challenging to draw if this representation would not be used. For y= (x-1)2 (x2-2x-1), the function's derivative become from the degree of three and resultantly'= x (4x2-12x+8). In the representation, we see critical points as 0, 1, and 2. The sign of the Y' is given in the last row of the table. We also see the function of the derivative sign in the last table as part of this multi representation. We could easily add those parts up and build a whole picture as an exclusive representation. Hence, the whole page becomes our systematic representation.

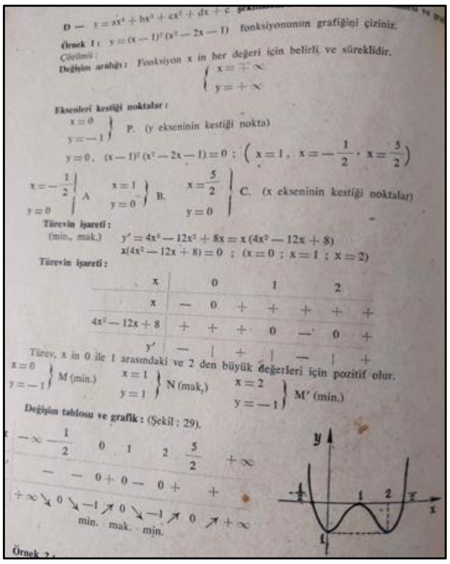
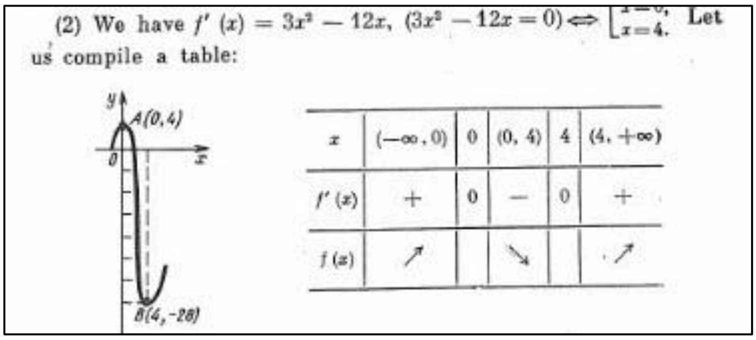


Figure 6. Akçabay(1961) divided but systematic p. 140

In Bogomolov (1986), (Figure 7) we see a similar part of the representation for y'= $3x^2$-12x, the table gets much more straightforward. However, we see that some parts of the representation are neglected. Another use of the same representation type X can be seen in Figure 8. This time, the aim is to show the solution to the inequality. Again most of the essential variables are used as critical points (especially indefinite ones as x= -7/4) and signs of the factors of the inequality function and the solution set as in the final row.



x	$(-\infty, 0)$	0	(0, 4)	4	$(4, +\infty)$
$f'(x)$	+	0	−	0	+
$f(x)$	↗		↘		↗

Figure 7. Bogomolov(1986)p.126

We have shared an essential but not highly recognized representation of type X with examples from the literature and some old course books. All these examples may say something about the non-problematic nature of the systematic structure of a semiotic representation as in the form of representation X. This kind of representation is more than one, as can be seen from those books at hand (Figure 9).

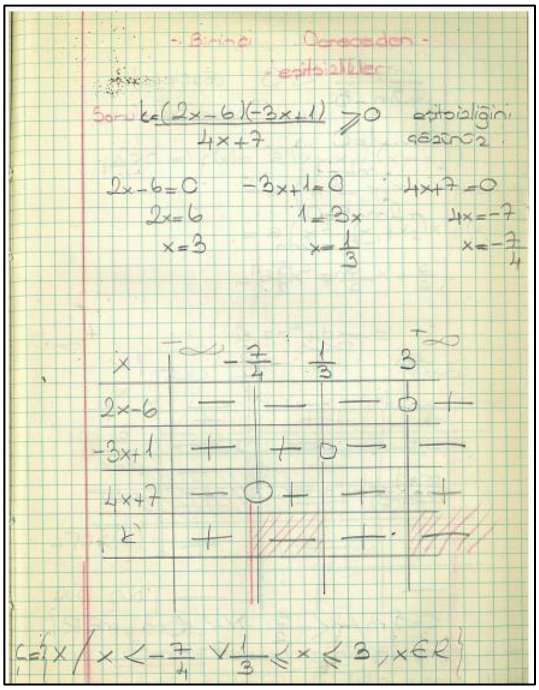


Figure 8. Same representation but for a different aim

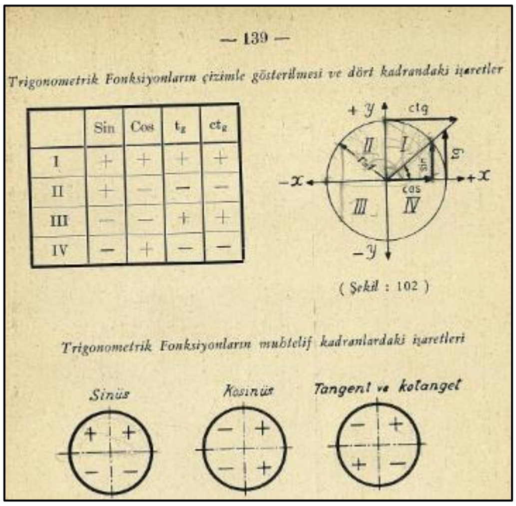


Figure 9. Baykara(1942) uses the same systematic but for trigonometric formulas and the unit circle.p.139

In Figure 9, we see a similar systematic representation for unit circle and trigonometric formulas. Signs of sin and cos specify regions. Tan and cotangent lines are specified by good drawing, and the way to detect the angle direction can be differentiated from the figure parts as above.

Table 1. Çanakçı (2006)

X	Y	IxI	IyI	IxI+IyI=2
x>0	y>0	x	y	x+y=2
x<0	y>0	-x	y	-x+y=2
x<0	y<0	-x	-y	-x-y=2
x>0	y<0	x	-y	x-y=2

In Table 1, one can see another type of this systematic representation as of Çanakçı (2006). Here step by step analysis of the absolute value function becomes easier to draw and its formation from the signs of the x and ys. Each row is a region in the graph. Hence, graphing becomes an interactive phase with necessary back and forth movements between the signs of x and y and the functions' behavior.

Thomas, Weir, Hass, & Heil (2014) uses a new kind of representation for the same representation X. One could argue that this is a much less systematic version of it (Figure 10). In a way, it may say that we are losing these semiotic representations like structures from textbooks since we did not give enough attention to them on time. In this new version, the increasing or decreasing behavior of the function is emphasized, but the critical points are neglected due to their role in intervals. Again the sign of f' is specified, but it is now explained in much detail as we already have seen above in representation X.

Solution The function f is continuous since $f'(x) = 4x^3 - 12x^2$ exists. The domain of f is $(-\infty, \infty)$, and the domain of f' is also $(-\infty, \infty)$. Thus, the critical points of f occur only at the zeros of f'. Since

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3),$$

the first derivative is zero at $x = 0$ and $x = 3$. We use these critical points to define intervals where f is increasing or decreasing.

Interval	$x < 0$	$0 < x < 3$	$3 < x$
Sign of f'	$-$	$-$	$+$
Behavior of f	decreasing	decreasing	increasing

Figure 10. Revision to this representation from Thomas' Calculus.p.223

3.2. Discussion

It was an urge to look for similar representations of this kind in the literature. We ended up with two representations: one with unit circle and trigonometric relation and the other for the absolute value function graphing. We do believe that these two representations are systematic. If you were taught these representations, you would probably never forget these two topics. Let's analyze these two representations from the point of s, view of Maani & Maharaj (2002), the performers' approach. We can never predict. These two representations are dynamic (timely in order), cover plausible explanations for the topic, enable the performers to see the big picture, carry causality, and enclose closed-loop structures within. More than Maani & Maharaj (2002), they enable quantitative respect and are scientific. Hence, a performer would be successful if a better one displays cyclic learning with these representations to understand the structure beforehand. This parallels what Maani & Maharaj (2002) posited as the number of systems thinking correlated with understanding system structure, completed experience cycles with these structures, and seeing the big picture with those at hand. Similarly, Lee et al. (2018) posited that matrix-like tables help understand and reduce cognitive overload.

Jung (2010) suggested this may help reduce the use of working memory capacity. Here as he suggested, the material is sequenced from simple to complex, and attention is split in parts rather than the whole picture at once, redundant information is shared in different nodes of representation, cause and effect analysis, not squashing to the representation material and focusing on learning such as interpreting, exemplifying, classifying, inferring, differentiating, and organizing in the instruction. All three cognitive load aspects are there: intrinsic, extraneous and germane. But it does not cause a problem since it is well structured and analyzed accordingly.

Sweller et al (1998) & Aditomo (2009) pointed out that the non-controversial and widely accepted structured representations may help, as in the example shown in this article, and the need for direct instruction. They may even help produce class discussions, which is helpful (Richland, et. al. 2017). Finally, Russo & Hopkins (2017) stress the need for challenging tasks for the whole class to increase intrinsic cognitive load and reduce extraneous cognitive load.

CONCLUSION

In conclusion, there are some representation types that we should never neglect. Representation X is one of these. Another similar example is a unit circle and trigonometry representation that we may call representation Y. We should carry them to new textbooks, in new course notes, and we should never forget to discuss them with students. We recommend detecting, finding, and searching for what works from old times and what does not work to give them their deserved status in our curriculum. In Finland, their curriculum's special attention is that they change some things in 100 years, not in 5 years. I believe this is important. Sometimes, when we change something to change them, we lose some particular points. Moreover, understanding what systematic relationships make this representation X noteworthy may help some IDs (interactive diagrams) and some mathlets to be a fighter with the cognitive overload as students of today suffer. In conclusion, it is very strategic not to lose some representations for the sake of new ones if their value is already known, especially about systematicity and not discussed too much.

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