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A Stochastic Integrated Inventory Model Single Supplier-Single Retailer in Periodic Review with Losing Flexibility Cost

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Abstract. *Efficiency in the supply chain can be established by integrating the supplier-retailer inventory policy. This article proposes the integrated inventory model between supplier-retailer under stochastic demand. This model aims to determine the optimal review period and calculate the total inventory cost, which contains some defective items, backorder price discounts and losing flexibility cost . We assume that the retailer can order 'n' times for every 'm' shipment from supplier to retailer in each production cycle under a periodic review. Stochastic conditions can cause sudden changes in orders by the retailer in large quantities, eventually forcing the supplier to reduce their setup policies. This condition makes the retailer be charged a losing flexibility cost as compensation for the reduction setup pushed by the supplier in a long-term partnership contract. Based on numerical examples and sensitivity analysis, the percentage of defective items in each shipment from supplier to retailer significantly affects integrated total inventory cost.*

Keywords: *integrated inventory model; losing flexibility cost; periodic review; set up policy; stochastic*

I. INTRODUCTION

Technological developments and business demand make inventory policy no longer focus on looking at one side only but on how to create the integration between many parties in the supply chain network. The purpose of the integrated inventory is to increase effectiveness and efficiency in the supply chain. Goyal, (1977) was the first author who investigated and proposed the integrated inventory model. This model significantly reduces the total inventory cost between supplier and customer. This model was later developed by Banerjee (1986) for a vendor with limited production rates and fixed delivery size. Later, Goyal (1988) improve his first model by changing the assumption production batch as a positive integer multiple of the retailer quantity order. Furthermore, Goyal (1995) extends the previous model by assuming that shipments can be carried out in different quantities under a continuous review policy. The continuous review policy assumes that the quantity of items in the warehouse is equal to the items in a deterministic system, so the order lot size will always remain, with times between two orders varying. This concept is different from a periodic review, where the times between two orders are always fixed with varying order lot sizes.

Many research that related to the integrated inventory model adopted a continuous review policy and only a few research that followed the periodic review policy. Lin (2010) who develop an integrated inventory model under periodic review. This model considers the existence of defective items, backorder price discounts, and variable lead time. Reducing the lead time (lead time reduction) can be an alternative solution to minimize the total inventory cost in the supply chain. The way to reduce the lead time is by adding crashing cost as a cost component on the buyer side. Lin and Lin (2016) develop a similar model with a recovery process for defective items. Mayangsari et al., (2017) propose an integrated model with adjusted production rates and variable lead time. Lin (2015), Kurdhi (2016) and Kurdhi and Doewes (2019) develop Lin (2010) model by assuming that the lead time and ordering cost are independent under a periodic review policy. Jauhari (2016) are also develop the integrated inventory model between vendor-buyer that contain defective items. Furthermore,

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Jauhari et al., (2017) considered the factor of human error when checking defective items that are received on every shipment by the supplier. Later, Hojati et al., (2017) proposed a delay in payment contracts in an integrated inventory model with stochastic demand, which was improved by Ebrahimi et al., (2019) and Nouri et al., (2021) where stochastic demand was assumed to depend on the promotional effort.

Lead time reduction is not the only way to minimize the total inventory cost on the supply chain. The way to minimize the total cost can also be done by implementing setup cost reduction. Jauhari and Saga (2017) developed an integrated model that seeks to reduce the setup cost in case of imperfect production and inspection. This model assumes that the vendor has the opportunity to spend some money or resource (invest) as a way to reduce setup costs. Castellano et al., (2017) proposed the integrated inventory model with investment as a strategy to reduce setup/ ordering cost and improve quality by considering the backorder price discount and variable lead time. Kurdhi et al., (2017) propose an integrated model considering variable lead time, setup cost reduction, and service level constraint. This model shows that the higher service level significantly influences customer loyalty which is crucial to building a competitive advantage in the market. Huang & Song (2020) also developed an integration model with service level constraints, but the periodic review policy happens to a single vendor-multi buyer.

In the real world, investment is not always the best solution to minimize the setup cost. Setup cost reduction can also be done by adding a losing flexibility cost on the buyer side. Losing flexibility cost is a losing cost of buyer flexibility to switch to another supplier or product in a long-term contract. Stochastic conditions can cause sudden demand by buyers in large quantities. The demand for certain types of products can increase during times such as christmas or new year. Stochastic conditions can cause sudden demand by buyers in large quantities. The demand for certain types of products can increase during times such as Christmas and new year. An increase in demand causes a buyer to force the supplier to reduce their setup time so that the buyer does not lose the opportunity to earn profit. This condition causes the buyer to be charged with losing flexibility cost as a dispensation for reduction setup made by a supplier. Considering losing flexibility cost as a parameter in an integrated model first time investigated by Kelle et al., (2003) and also contained in the Linarti (2014) model that shows a decrease in the joint total inventory cost in the supply chain.

Based on some research above, we can see that the periodic review policy has been widely considered in the recent model development, but none of them has involved losing flexibility cost. Referring to Lin (2010) model, we proposed an integrated inventory model with stochastic demand between supplier-retailer and considering defective items, backorder price discount, and losing flexibility cost under periodic review policy.

II. RESEARCH METHOD

Notation

The notation that used in this model is :

$\boldsymbol{T}$: review period (year)
$\boldsymbol{D}$: average demand rate (units/ year)
$\boldsymbol{P}$: production rate (units/ year)
$\boldsymbol{S}$: setup cost ($/ setup)
$\boldsymbol{L}$: lead time (week)
$\boldsymbol{\sigma}$: standard deviation of the demand (units/ year)
$\boldsymbol{v}$: production cost per unit
$\boldsymbol{R}$: target level
$\boldsymbol{p}$: unit purchasing cost
$\boldsymbol{s}$: unit inspecting cost
$\boldsymbol{x}$: inspecting rate
$\boldsymbol{\gamma}$: probability of defective items, $0< \gamma<1$
$\boldsymbol{\omega}$: unit treatment cost of defective items
$\boldsymbol{h_s}$: holding cost for supplier ($/ units/ year)
$\boldsymbol{h_{r1}}$: holding cost of non-defective items for retailer ($/ units/ year)
$\boldsymbol{h_{r2}}$: holding cost of defective items for retailer ($/ units/ year)
$\boldsymbol{\beta}$: backorder ratio, $\boldsymbol{0 < \beta < 1}$
$\boldsymbol{\beta 0}$: upper bound of the backorder ratio
$\boldsymbol{\pi x}$: backorder price discount per unit

$\pi 0$: marginal profit per unit
l : losing flexibility rate
k : safety factor
m : number of deliveries
n : number of orders
X : protection interval

Assumptions

The assumption that used in this model is:

1. This model considers the relationship between a single supplier-single retailer with a single product in a long-term contract.
2. Retailer apply a periodic review policy to manage their inventory.
3. A stochastic demand follows a normal distribution with a standard deviation σ
4. For each DT unit of retailer orders, the supplier will produce mDT units in each production cycle with a production rate P.
5. The inventory level is reviewed for every T unit of times. A sufficient quantity is ordered up to the target level R and arrives after L units of time.
6. Lead time L is a constant parameter and does not exceed the review period T ($L < T$), but it is allowed to place an order more than once per cycle.
7. Target level R = expected demand during the protection interval + safety stock or $R = D(T+L) + k\sigma\sqrt{T+L}$, where k represent the value of the safety factor.
8. The retailer can order n times, and suppliers deliver m times for one set up with a limited production rate P.
9. Every order received by the retailers is assumed to contain some defective items with probability γ, so the retailer will inspect to check those defective items.
10. Shortages are allowed in retailers and apply partial backorder. Retailers offer discounts to loyal customers and reserve the right to consider the offer if the value of πx is better than $\pi 0$.

Model Development

In this model, the retailer is assumed to be able to place an orders 'n' times, and the supplier delivers in 'm' times in a cycle T. For each DT units of retailer orders, the supplier will produce mDT units in each production cycle, so the length of each ordering cycle for the retailer is T/n and mT is the length of each production cycle for the supplier. The value of the number of orders n and delivery m satisfies $m = n$ or $m \neq n$.

The inventory level of retailer and supplier can be seen in Figure 1.

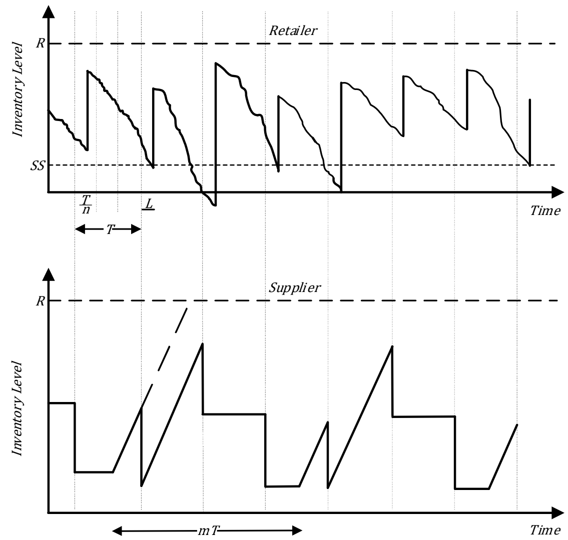

Figure 1. Inventory level for supplier and retailer

Supplier's Inventory Cost

For each production cycle, the setup cost per unit incurred by the supplier is S/mT. The supplier will produce the first DT unit and immediately deliver it to the retailer, and deliver on average every T unit of time for the next delivery until the inventory level runs out. We can calculate the average retailer inventory per unit time by following the equation :

$$=\frac{DT}{2}\left(\frac{D}{P}(2-m)+(m-1)\right) \tag{1}$$

Hence, the supplier holding cost is :

$$=hs\frac{DT}{2}\left(\frac{D}{P}(2-m)+(m-1)\right) \tag{2}$$

It's clear that the supplier produces some defective items from the production process as much as γDT unit in each cycle T. The supplier will be charged a treatment cost of defective items per unit ω, so the total treatment cost per unit time is $\omega\gamma D$. Therefore, we can calculate the total cost from a supplier by adding all cost above, which is production cost, setup cost, holding cost, and treatment cost of defective items, is expressed by :

TCs $=Dv + \frac{S}{mT} + hs\frac{DT}{2}\left(\frac{D}{P}(2-m) + (m-1)\right) + \omega\gamma D$ (3)

Retailer's Inventory Cost

Retailers implement periodic review policies to manage their inventory. It's assumed that in each cycle T, the retailer place n times of orders with the length of ordering cycle T/n. As previously explained, there are two kinds of items that retailer receive from a supplier: defective items with defective rate γ and non-defective items $(1-\gamma)DT$. The retailer will inspect the total product with an inspection rate x, and the inspection cost per unit is s. Hence, the total inspection cost per unit time is sD for each DT unit received. The retailer will first store defective items findings and return them to the supplier on the next shipment. This condition causes the retailer to incur holding cost for defective items and non-defective items. Total holding cost for non-defective items can be writen as follows :

$$=\frac{hr1}{T}\left(\frac{DT^2}{2} + \frac{\gamma(DT)^2}{2x}\right) \quad (4)$$

Where h_{r2} is the holding cost for defective items per unit, then the total holding cost for defective items is :

$$=\frac{h_{r2}}{T}\left(\gamma DT^2 - \frac{\gamma(DT)^2}{2x}\right) \quad (5)$$

Therefore, the total holding cost for both types of items received by the retailer is expressed by :

$$= DT\left(\frac{h_{r1}}{2} + h_{r2}\gamma + \frac{\gamma D}{2x}(h_{r1} - h_{r2})\right) \quad (6)$$

We assumed that the interval protection (review period + lead time) demand X follows a normal distribution with mean $D(T+L)$ and standard deviation $k\sigma\sqrt{T+L}$, target level $R = D(T+L) + k\sigma\sqrt{T+L}$. A shortage is allowed in the retailer by implementing backorder under a periodic review policy. We know that the backorder ratio is β, and the retailer's backorder price discount is πx per unit, so $\beta = \beta 0\pi x/\pi 0$, where $0 \le \beta 0 \le 1$, and $0 \le \pi x \le \pi 0$. The expected shortage cost per year is $\frac{1}{T}\left(\pi x\beta + \pi 0(1-\beta)\right)E(X-R)^+$, with $E(X-R)^+$ is the expected shortage quantity at the end of the cycle and expressed as :

$$=\int_R^\infty (x-R)fx(x)dx = \sigma\sqrt{T+L}\psi(k) > 0 \quad (7)$$

where: $\psi(k) = \phi(k) - k\left(1-\Phi(k)\right)$, $\phi(k)$ is a standard normal probability density function, dan $\Phi(k)$ is a standard normal cumulative density function.

Thus, the overall retailer inventory cost is the total of several cost consisting of purchase cost, ordering cost, holding cost of the defect and non-defect items, backorder costs, inspection costs, and losing flexibility costs, or mathematically it can be written:

$$TCr = \frac{pDT}{T} + n\frac{A}{T} + \frac{DT}{n}\left(\frac{hr1}{2} + hr2\gamma + \frac{\gamma D}{2x}(hr1 - hr2)\right) + n\frac{1}{T}\left(\frac{\beta 0\pi x^2}{\pi 0} + \pi 0 - \beta 0\pi x\right)\sigma\sqrt{T+L}\psi(k) + sD + \left(\frac{DT}{2n} + k\sigma\sqrt{\frac{T}{n}+L}\right)lp \quad (8)$$

Joint Total Inventory Cost

We consider that supplier and retailer have agreed to determine the best integrated periodic review policy strategy. Thus, the joint total inventory cost under the integrated policy is the sum of supplier inventory cost + retailer inventory cost, expressed as :

$$\text{TC} = Dv + \frac{S}{mT} + hs\frac{DT}{2}\left(\frac{D}{P}(2-m) + (m-1)\right) + \omega\gamma D + \frac{pDT}{T} + n\frac{A}{T} + \frac{DT}{n}\left(\frac{hr1}{2} + hr2\gamma + \frac{\gamma D}{2x}(hr1 - hr2)\right) + n\frac{1}{T}\left(\frac{\beta 0\pi x^2}{\pi 0} + \pi 0 - \beta 0\pi x\right)\sigma\sqrt{T+L}\psi(k) + sD + \left(\frac{DT}{2n} + k\sigma\sqrt{\frac{T}{n}+L}\right)lp$$ 9)

The goal of developing this periodic review model is to find the optimal value T, πx, k, m, and n, so $TC(T, \pi x, k, m, n)$ reaches the minimum value. For fixed m and n, taking the first partial derivatives of $TC(T, \pi x, k, m, n)$ respect to T, πx, and k using software Maple 2020. So that we get the following equation :

$$\frac{\partial TC(T,\pi x,k,m,n)}{\partial k}=0$$

$$Fs(k)=1-\frac{lpT}{n\left(\frac{\beta 0\pi x^2}{\pi 0}+\pi 0-\beta 0\pi x\right)} \quad (10)$$

$$\frac{\partial TC(T,\pi x,k,m,n)}{\partial \pi x}=0$$

$$\pi x=\frac{\pi 0}{2} \quad (11)$$

$$\frac{\partial TC(T,\pi x,k,m,n)}{\partial T}=0$$

$$T=\left(\left(-\frac{S}{m}-An-n\left(\frac{\beta 0\,\pi^2 x}{\pi 0}+\pi 0-\beta 0\,\pi x\right)\sigma\sqrt{\frac{T}{n}+L}\,\psi(k)\right)\Bigg/\left(-\frac{hs\,D\left(\frac{D\,(2-m)}{P}+m-1\right)}{2}-\frac{D\left(\frac{hr1}{2}+hr2\,\gamma+\frac{\gamma D\,(hr1-hr2)}{2\,x}\right)}{n}-\frac{\left(\frac{\beta 0\,\pi^2 x}{\pi 0}+\pi 0-\beta 0\,\pi x\right)\sigma\,\psi(k)}{2\,T\sqrt{\frac{T}{n}+L}}-\left(\frac{D}{2\,n}+\frac{k\sigma}{2\sqrt{\frac{T}{n}+L}\,n}\right)lp\right)\right)^{1/2} \quad (12)$$

To find the value of T^*, πx, k^*, m^*, and n^* It cannot be done directly, considering that each of these parameters depends on each other. For example, to get the value of T, we must first know the value of k. So we need an iterative method to get the optimal solution to the problem as follows:

1. Set $m=1$
2. For every $n=m$ or $n\neq m$, with constant lead time L, and $\pi x=\pi 0/2$, then the value of T can be determined from equation (12)
3. Use the value of T to get the value of k from equation (10)
4. Perform calculations until there is no change in values of T and k
5. Set $T^*_{mn}=T$, and $k^*_{mn}=k$
6. Compute joint total cost $TC(T^*_{mn},\pi x,k^*_{mn},m,n$ from equation (9)
7. If $TC(T_{mn},\pi x,k_{mn},m,n)\leq TC(T^*_{((m-1)n)},\pi x,$ $k^*_{((m-1)n)},(m-1),n)$ repeat steps 2-7 with $m=m+1$, but if not then go to step 8
8. Set $TC(T^*,\pi x,k^*,m^*,n^*)$=$TC(T^*_{((m-1)n)},$ $\pi x,k^*_{((m-1)n)}$, so $TC(T^*,\pi x,k^*,m^*,n^*)$ is the minimum joint total inventory cost, and $(T^*,\pi x,k^*,m^*,n^*)$ is the optimal solution.

III. Result and Discussion

Numerical Example

To illustrate the algorithm above, we adopt the data used in the research of Ouyang et al. (2007) by adding several parameters needed in the proposed model..

$\boldsymbol{D}$:	600 unit/ year	$\boldsymbol{h_{r2}}$:	\$25/ unit/ year
$\boldsymbol{A}$:	\$200/ order	$\boldsymbol{x}$:	1500
$\boldsymbol{P}$:	1200 unit/ year	$\boldsymbol{s}$:	\$0.5/ unit
$\boldsymbol{S}$:	\$300/ order	$\boldsymbol{\omega}$:	\$30/ unit
$\boldsymbol{\pi 0}$:	\$150/ unit	$\boldsymbol{l}$:	0.03
$\boldsymbol{\sigma}$:	7 unit/ week	$\boldsymbol{L}$:	2 week
$\boldsymbol{v}$:	\$60/ unit	$\boldsymbol{\gamma}$:	0,005-0,045
$\boldsymbol{p}$:	\$65/ unit	It is assumed that one year = 52 weeks	
$\boldsymbol{h_s}$:	\$20/ unit/ year		
$\boldsymbol{h_{r1}}$:	\$30/ unit/ year		

The optimal result is shown in Table 1 to 5. We can see that the minimum total inventory cost under the integrated model occurs at conditions $m=1$ and $n=2$ for $\beta 0=1$, $\gamma=0.005$, and $k=2.815$. The total cost from supplier, retailer, and joint inventory cost is respectively, \$38,000.03, \$42,604.03, and \$80,604.06. Based on the results in Table 1 to 5, we can observe that the increase in the percentage of defective items γ, also increases the joint total inventory cost. Contrary to the percentage of defective item γ, an increase in the value of the backorder ratio $\beta 0$ causes a decrease in the joint inventory cost. Another

Table 1. An optimal solution for $\beta 0 = 0$ (T^* in weeks and TCs, TCr, TC in $)

γ	T^*	πx^*	k^*	m^*	n^*	$TCs(T^*, m^*)$	$TCr(T^*, \pi x^*, k^*, n^*)$	$TC(T^*, k^*, \pi x^*, m^*, n^*)$
0,005	14,61	75	2,907	1	2	38000,58	42615,77	80616,35
	7,31	75	2,907	2	1	38000,58	42615,77	80616,35
0,015	14,55	75	2,908	1	2	38181,61	42637,16	80818,76
	7,27	75	2,908	2	1	38181,61	42637,16	80818,76
0,025	14,48	75	2,909	1	2	38362,67	42658,41	81021,08
	7,24	75	2,909	2	1	38362,67	42658,41	81021,08
0,035	14,42	75	2,911	1	2	38543,75	42679,53	81223,29
	7,21	75	2,911	2	1	38543,75	42679,53	81223,29
0,045	14,36	75	2,912	1	2	38724,87	42700,53	81425,40
	7,18	75	2,912	2	1	38724,87	42700,53	81425,40

Tabel 2. An optimal solution for $\beta 0 = 0,3$ (T^* in weeks and TCs, TCr, TC in $)

γ	T^*	πx^*	k^*	m^*	n^*	$TCs(T^*, m^*)$	$TCr(T^*, \pi x^*, k^*, n^*)$	$TC(T^*, k^*, \pi x^*, m^*, n^*)$
0,005	14,62	75	2,882	1	2	38000,43	42612,61	80613,04
	7,31	75	2,882	2	1	38000,43	42612,61	80613,04
0,015	14,56	75	2,883	1	2	38181,46	42634,01	80815,47
	7,28	75	2,883	2	1	38181,46	42634,01	80815,47
0,025	14,49	75	2,885	1	2	38362,51	42655,28	81017,79
	7,25	75	2,885	2	1	38362,51	42655,28	81017,79
0,035	14,43	75	2,886	1	2	38543,59	42676,41	81220,00
	7,21	75	2,886	2	1	38543,59	42676,41	81220,00
0,045	14,37	75	2,888	1	2	38724,70	42697,42	81422,12
	7,18	75	2,888	2	1	38724,70	42697,42	81422,12

Table 3. An optimal solution for $\beta 0 = 0,6$ (T^* in weeks and TCs, TCr, TC in $)

γ	T^*	πx^*	k^*	m^*	n^*	$TCs(T^*, m^*)$	$TCr(T^*, \pi x^*, k^*, n^*)$	$TC(T^*, k^*, \pi x^*, m^*, n^*)$
0,005	14,63	75	2,855	1	2	38000,27	42609,17	80609,44
	7,32	75	2,855	2	1	38000,27	42609,17	80609,44
0,015	14,57	75	2,856	1	2	38181,29	42630,58	80811,87
	7,28	75	2,856	2	1	38181,29	42630,58	80811,87
0,025	14,50	75	2,858	1	2	38362,34	42651,86	81014,20
	7,25	75	2,858	2	1	38362,34	42651,86	81014,20
0,035	14,44	75	2,859	1	2	38543,42	42673,01	81216,42
	7,22	75	2,859	2	1	38543,42	42673,01	81216,42
0,045	14,38	75	2,861	1	2	38724,52	42694,03	81418,55
	7,19	75	2,861	2	1	38724,52	42694,03	81418,55

exciting thing we can observe is that for every number of m shipments or n orders that remain unchanged under backorder ratio $\beta 0$, the value of T decreases for every increase in the percentage of defective item γ.

A comparison of calculation between the independent and the integrated model by considering losing flexibility cost is shown in Table 6.

Based on Table 6, we can observe that the total joint inventory cost comparison between the independent and the integrated model is not very significant. It can be seen from the difference of no more than 1%. Similar results were also found in the research of Lin (2010), Kurdhi (2016), and Mayangsari et al., (2017). This shows an indication that the implementation of joint inventory cost in the periodic review policy results in insignificant savings.

Table 4. An optimal solution for $\beta 0 = 0,9$ (T^* in weeks and TCs, TCr, TC in \$)

γ	T^*	πx^*	k^*	m^*	n^*	$TCs(T^*, m^*)$	$TCr(T^*, \pi x^*, k^*, n^*)$	$TC(T^*, k^*, \pi x^*, m^*, n^*)$
0,005	14,65	75	2,825	1	2	38000,09	42605,38	80605,47
	7,32	75	2,825	2	1	38000,09	42605,38	80605,47
0,015	14,58	75	2,827	1	2	38181,11	42626,81	80807,91
	7,29	75	2,827	2	1	38181,11	42626,81	80807,91
0,025	14,52	75	2,828	1	2	38362,15	42648,10	81010,25
	7,26	75	2,828	2	1	38362,15	42648,10	81010,25
0,035	14,45	75	2,830	1	2	38543,22	42669,26	81212,49
	7,23	75	2,830	2	1	38543,22	42669,26	81212,49
0,045	14,39	75	2,831	1	2	38724,32	42690,30	81414,62
	7,19	75	2,831	2	1	38724,32	42690,30	81414,62

Table 5. An optimal solution for $\beta 0 = 1$ (T^* in weeks and TCs, TCr, TC in \$)

γ	T^*	πx^*	k^*	m^*	n^*	$TCs(T^*, m^*)$	$TCr(T^*, \pi x^*, k^*, n^*)$	$TC(T^*, k^*, \pi x^*, m^*, n^*)$
0,005	14,65	75	2,815	1	2	38000,03	42604,03	80604,06
	7,33	75	2,815	2	1	38000,03	42604,03	80604,06
0,015	14,58	75	2,816	1	2	38181,04	42625,46	80806,51
	7,29	75	2,816	2	1	38181,04	42625,46	80806,51
0,025	14,52	75	2,818	1	2	38362,08	42646,76	81008,85
	7,26	75	2,818	2	1	38362,08	42646,76	81008,85
0,035	14,46	75	2,819	1	2	38543,15	42667,93	81211,08
	7,23	75	2,819	2	1	38543,15	42667,93	81211,08
0,045	14,39	75	2,820	1	2	38724,25	42688,97	81413,22
	7,20	75	2,820	2	1	38724,25	42688,97	81413,22

Table 6. Independent policy and integrated policy for $\gamma = 0.005$

Model	m	n	$T^*(weeks)$	TCs (\$)	TCr (\$)	TC (\$)
Independent	3	1	6,77	38029,83	42616,27	80646,09
Integrated	2	1	7,31	38000,58	42615,77	80616,35
	1	2	14,61	38000,58	42615,77	80616,35

Furthermore, using the same data and assumptions, sensitivity analysis can be performed to determine the effect of changes in a few parameters on the results of the integrated model. Each parameter was changed to -25% and +25% from the initial value, assuming $\beta 0 = 0.6$ and $\gamma = 0,025$ for sensitivity analysis. The results of the sensitivity analysis are shown in Table 7.

Based on Table 7 above, we can observe :

1. An increase in D causes an increase in the joint total inventory cost $TC(T^*, \pi x, k^*, m^*, n^*)$ along with a decrease in the value of T^*. This result shows that the value of T^* and the total cost of $TC(T^*, \pi x, k^*, m^*, n^*)$ is very sensitive to changes in parameter D.
2. An increase in A and S causes an increase in review period T^*, but it doesn't affect the joint total inventory cost $TC(T^*, \pi x, k^*, m^*, n^*)$ significantly. This result shows that the value of T^* is lowly sensitive to changes in parameter A and S.
3. Changes in the value of P and h_{r2} don't affect the value of T^* and joint total cost $TC(T^*, \pi x, k^*, m^*, n^*)$.
4. The increase in the value of v doesn't affect the value of T^*, but significantly causes the increase in joint total cost $TC(T^*, \pi x, k^*, m^*, n^*)$ because the supplier must spend more.
5. The increase in the value h_s doesn't significantly affect the joint total cost $TC(T^*, \pi x, k^*, m^*, n^*)$, but is lowly sensitive to changes in the value of T^*.

Table 7. Effect of parameter changes on the proposed model

Parameter	% change	m	n	πx	T	k	TCs	TCr	TC
D	-25	2	1	0	14%	-1%	-24,4%	-23,9%	-24,1%
	+25	2	1	0	-10%	1%	24,3%	23,8%	24,0%
A	-25	2	1	0	-8%	1%	0,1%	-0,9%	-0,4%
	+25	2	1	0	7%	-1%	0,0%	0,8%	0,4%
$\pi 0$	-25	2	1	-0,25	0%	-3%	0,0%	0,0%	0,0%
	+25	2	1	0,25	0%	3%	0,0%	0,0%	0,0%
P	-25	2	1	0	0%	0%	0,0%	0,0%	0,0%
	+25	2	1	0	0%	0%	0,0%	0,0%	0,0%
S	-25	2	1	0	-6%	1%	-0,7%	0,0%	-0,3%
	+25	2	1	0	5%	-1%	0,6%	0,0%	0,3%
v	-25	2	1	0	0%	0%	-23,5%	0,0%	-11,1%
	+25	2	1	0	0%	0%	23,5%	0,0%	11,1%
h_s	-25	2	1	0	5%	0%	-0,6%	0,0%	-0,3%
	+25	2	1	0	-4%	0%	0,6%	0,0%	0,3%
h_{r1}	-25	2	1	0	7%	-1%	0,0%	-0,8%	-0,4%
	+25	2	1	0	-6%	1%	0,0%	0,7%	0,4%
h_{r2}	-25	2	1	0	0%	0%	0,0%	0,0%	0,0%
	+25	2	1	0	0%	0%	0,0%	0,0%	0,0%

Table 8. Effect of changes in parameters D and A on the proposed model

Parameter	m	n	πx	T^*(week)	k	TCs	TCr	TC
D-, A-	2	1	0	5,15%	-0,56%	-24,35%	-24,67%	-24,52%
D-, A+	2	1	0	22,03%	-2,23%	-24,44%	-23,15%	-23,76%
D+, A-	2	1	0	-16,59%	2,00%	24,40%	22,77%	23,54%
D+, A+	2	1	0	-3,37%	0,38%	24,30%	24,69%	24,51%

6. Increasing the value of h_{r1}causes a decrease in the value of T^* but doesn't significantly affect the joint total cost $TC(T^*, \pi x, k^*, m^*, n^*)$.

We can observe that the value of T^* is sensitive to changes in parameters D and A. It is interesting to take another sensitivity analysis to see the effect of changes in these two parameters in an integrated model. The results of this sensitivity analysis can be seen in Table 8.

Based on the sensitivity analysis above, we can see that the joint total inventory cost increased and decreased by about 23% over four different conditions. In addition, it is also seen that the value of T has increased by about 22% in reduced demand-increased order cost, and decreased by about 16% in the opposite condition. It is proven that changes in parameters D and A have a significant effect on the value of T^* and joint tocal inventory cost $TC(T^*, \pi x, k^*, m^*, n^*)$.

IV. CONCLUSION

This study develops a supplier-retailer integrated inventory model by considering the presence of defective items, backorder price discounts, and losing flexibility costs under a periodic review policy for stochastic demand. The results of the numerical example and sensitivity analysis show that the percentage of defective items contained in each shipment by the supplier to the retailer affects the joint total cost in the integrated model. Therefore, suppliers must try to reduce the percentage of defective items from each production process. The backorder price discount offered by retailers makes customers prefer to wait. Moreover, the addition of losing flexibility costs to retailers can be an alternative solution to reducing setup costs other than investment.

Future research can add the inspection error on the supplier or both sides. The limitations of the integrated model, which only discusses the relationship between single supplier-single retailers, can be developed for integration between single supplier-many retailers. In addition, considering multiple items can be included as a new parameter in the integration model.

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