

Knowledge of Student's Understanding and The Effect on Instructional Strategies: a Case of Two Novice Mathematics Teachers

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Abstract. *Pedagogical content knowledge plays a role in learning in classroom. The study aims to 1) analyze the teachers' knowledge of the students' errors and difficulties in solving linear equations and 2) describe the instructional strategies used by teachers to reduce their errors and difficulties. The subjects were two novice teachers who have taught the Junior High School 7th grade students with different math abilities for two years. The data collection was conducted by open questionnaire and interview. The results showed that both teachers have knowledge of students' errors and difficulties in solving linear equations. However, the teacher's learning strategies in helping students reduce students' errors and difficulties were different from one another. It can be concluded that the teacher's knowledge of the students' errors and difficulties can help teachers determine the appropriate learning strategies to present a learning subject matter.*

Keywords: *pedagogical content knowledge, students' errors and difficulties, teachers' strategies*

Introduction

Teaching is not an easy job. Teachers need a variety of knowledge to be able to teach well. As stated by NCTM (2000), "teaching mathematics well is a complex endeavor, and there are no easy recipes" (p.17). Furthermore, NCTM (2000) states that for an effective learning process, a teacher must have in-depth knowledge and understanding of the content, curriculum, student characteristics, learning methods, and classroom management. A comprehensive study of the knowledge of teachers has been discussed by Shulman (1986). In his study of an effective teaching process, Shulman stated that there are three kinds of the teachers' knowledge: content knowledge (CK), pedagogical content knowledge (PCK), and curricular knowledge. PCK is defined by Shulman (1986) as "content knowledge that embodies the aspects of content most germane to its teachability" (p.6). Furthermore, Shulman (1987) developed the PCK definition as "representing the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction" (p. 8).

In addition to the knowledge to present the content, PCK includes knowledge of students' understanding, the purpose of learning content, understanding students'

difficulties in learning topics, conceptions and misconceptions, knowledge of learning strategies, and student backgrounds that influence learning such as culture, and physical environment (Cochran, DeRuiter, & King, 1993; Grossman, 1989; Magnusson, Krajcik, & Borko, 1999; Marks, 1990; Shulman, 1986; Smith & Neale, 1991; Tamir, 1988). Grossman & Richert (1988) defines the knowledge of students' understanding as knowledge of the conceptions and misconceptions of specific subject-matter, initial understanding and students' knowledge of subjects, and student interest in the content. An et. al (2004) identified student misconceptions as a sub-category of one of the PCK components.

Some researchers have identified several subcategories of knowledge of students' understanding. Shulman (1986) stated that the knowledge of what causes an easy or difficult subject-matter for students as one of the characteristics of PCK, but he does not state it as a subcategory of knowledge of student's understanding. Furthermore, some scholars state that knowledge of students' errors on the subject-matter (Marks, 1990; Smith & Neale, 1991), student interest in subject-matter (Grossman & Richert, 1988; Park & Chen, 2012; Park & Steve Oliver, 2008), motivation (Cochran et al., 1993; Park & Chen, 2012; Park & Steve Oliver, 2008), knowledge of prerequisite content (Magnusson et al. 1999), and knowledge of student difficulties in studying subjects (Magnusson et al., 1999; Park & Chen, 2012; Park & Oliver, 2008) are part of knowledge of students' understanding.

In junior high school, students learn algebra by using variables to represent a quantity, solve an equation that contains variables, and represent the word problem in the form of algebraic expression. There are differences in the course of algebra in primary and junior high schools. In primary school, learning algebra focuses on the arithmetic operation by using numbers, but in junior high school, the course of algebra involves the variables in its operations. At this stage, students takes the cognitive transition of mathematical thinking from arithmetic to algebraic thinking (Fillooy & Rojano, 1989; Warren, 2003). In other words, the student's conception of mathematical operations has changed from numbers to variables (Fillooy & Rojano, 1989). Sometimes, the transition process does not work well. It causes the *cognitive gap*, the inability of students to perform operations using variables (Herscovics & Linchevski, 1994; Linchevski & Herscovics, 1996). As a result, students have difficulty in solving algebraic problems as linear equations of one variable.

In his study of 49 junior high school students, Papaieronymou (2007) concluded that students had difficulty in distinguishing algebraic forms from linear equations. The students assume that algebraic forms and linear equations are the similar expression. Nonetheless, they have no difficulty in manipulating or simplifying algebraic forms and solving linear equations. Magruder (2012) identified three difficulties of junior high school students in solving algebraic equations, i.e. understanding of symbols or variables, meaning of sign = (equal to), and emphasizing procedural knowledge without conceptual understanding. Brizuela & Schliemann (2004) explained the lack of students' understanding in solving algebraic problems. According to their research, the difficulties of students in learning algebra are due to low ability in arithmetic concepts and its application on algebra. Similarly, in the study of 51 Indonesian students in understanding algebra, Jupri, Drijvers, & Heuvel-Panhuizen (2014) concluded that students have difficulty in the process of mathematization, changing the word problem in mathematics expressions or symbols. Other difficulties include understanding algebraic expressions and application of arithmetic operations to algebra, and sign = (equal to) and variable.

The focus of the paper is on discussing the teachers' knowledge of the students' understanding of their errors and difficulties in studying linier equations of one variable.

It will also discuss the teachers' instructional strategy to teach algebra to them with different math ability.

Research Method

The research was conducted in Sukoharjo regency, Central Java Province, Indonesia. The subjects were two junior high school mathematics teachers (Joko and Ema) who have taught the different private Junior High Schools 7th grade students. They have the same education background as a bachelor degree of mathematics education. Joko has taught the students who are low in math ability, but Ema has taught those who are high in math ability.

The data was collected by using the interview and open questionnaires, containing four problems of linear equations of one variable. These were used for exploring their understanding of the student's errors. They were asked to write errors that are generally made by the students in solving the given problems. Furthermore, the interview was deeply intended to find out their understanding of the students' errors and their difficulties in solving linear equations of one variable. In addition, it was conducted to explore strategies used by teachers to teach algebra, especially solving linear equations of one variable.

Finding and Discussion

The teachers' answers to the students' errors in solving four questions of linear equations of one variable are reported in Table below.

Table
Teachers' answers to the student's errors

Number	Problem	Ema's answer	Joko's answer
1	$x + 10 = 47$	$x = 47 + 10$ $x = 57$	$x = 47 + 10$ $x = 57$
2	$4x - 1 = 0$	$4x = 1$ $x = 1: -4$ $x = -\frac{1}{4}$	$4x = 1$ $x = 1 - 4$
3	$2 - 4x = 3$	$-4x = 3 - 2$ $-4x = 1$ $x = \frac{1}{4}$	$4x = 3 - 2$ $4x = 1$ $x = 1 - 4$
4	$2 + \frac{x}{4} = 5$	$(2 + \frac{x}{4} = 5) \times 4$ $1 + x = 20$	$\frac{x}{4} = 5 - 2$ $x = 3$

In Table 1, there are Joko's and Ema's similar answers relating to the student's error in solving the first problem. According to the two teachers, the students mostly made misconceptions in solving $x + 10 = 47$. The answer to the first problem should be $x = 47 - 10 = 37$, but most of them answered $x = 47 + 10 = 57$. Based on the interview with the two teachers, the students were confusing with the concept of equality ($=$) in solving an equation. They did not understand every manipulation in an equation that the left side must be equal to the right one. This means that if the left side is added or subtracted by a number or variable, and so is the right side.

Furthermore, on the second and third problem, both the teachers gave different answers regarding the students' errors. According to Ema, the students had been able to understand the procedure to solve linear equations that contain coefficients on the variables, but they made misconceptions in solving equations. They understand that the concept of the operation of division on the linear equation is the same as the addition or subtraction. So, the divisor becomes -4 in the second answer and 4 in the third one.

On the other hand, Joko wrote down the similar answer to the student's errors on the second and third problems. According to him, the students did not understand the procedure to solve an equation that contains coefficients on the variable. They only understood the procedure on the addition or subtraction to solve the given problems. So, the students' answers to the linear equations problem of $4x = 1$ is $x = 1 - 4$.

The fourth problem is the linear equation where the variable x contains the coefficient of rational numbers. Ema stated that her students already understood the procedure to solve the given problem. The equation on the fourth problem was multiplied by the denominator of the rational number of the given problem, but some students made errors in multiplication. They did not multiply each term in the equation by the number of denominator. Frequently, they did not the calculations accurately. Otherwise, Joko stated that his students solved the fourth problem in a same way as the use of the procedure to solve the first one. They did not understand how to operate the equations with rational numbers. Thus, most of them ignored coefficients on variables in the form of rational numbers in solving linear equations.

Both of the teachers stated that the lack of the students' understanding of the rules on the arithmetic operation is the most influential factors that cause the emergence of the students' errors in solving algebraic problems especially linear equations. Further, based on the interviews, algebra is one of difficult subject matter to teach the junior high school students. The teachers have difficulty in determining strategies for teaching variables, constants, algebraic operations, and procedure to solve linear equations which can be understood by the students. The main difficulties relate to the differences in algebraic concepts which the students learn in primary and junior high school. Algebra in primary school emphasizes on operations with numbers or arithmetic, but the concept of algebra in junior high school focuses on operations with variables or abstract symbols.

Both of the teachers have the differences instructional strategies for teaching algebra especially in solving linear equations of one variable. Ema stated that for teaching the solution to the problem of linear equations which were understandable by students, she emphasized on conceptual learning. She taught the principle of equality on an equation before explaining the procedure to solve linear equations problems. To solve the first problem, for example, in the beginning she taught by using the principle of equality, i.e. each term (left and right sides) of equations is added with -10. Thus, the first problem becomes $x + 10 - 10 = 47 - 10$. Therefore the solution is $x = 37$. The students could solve the equation more easily if they understood conceptually in the first. In addition, the students would also ask the teacher if there are not conceptual-based procedures, such as "move-segment" operation often used by teachers to solve linear equations problems.

In contrast, Joko had difficulty in teaching conceptually in solving linear equations by students. He stated that they had difficulties in understanding the solution by using the equality principle of an equation. They are more likely a quick and practical procedure to solve a problem. Although they did not understand conceptually how the procedure is organized, for example a "move-segment" operation. According Joko, the students prefer remembering or memorizing to understanding the procedure. As a result, he emphasized

on providing exercises so that the students will be get accustomed to solving various types of problems.

In his study of ten primary and nine junior prospective school teachers, Ball (1988) found that teachers with deep understanding of subjects matter emphasized on conceptual learning. Likewise, Borko & Putnam (1996) in his study stated that teachers with deep-subject matter knowledge emphasized conceptual understanding, problem solving, and inquiry in teaching the subject matter. Conversely, the teachers with low understanding tended to emphasize on procedures, rules, facts, and sometimes forgot the important concepts and its relationships in learning. Rollnick, Bennett, Rhemtula, Dharsey, & Ndlovu (2008) in the researches of science teachers in South Africa also found that the teachers with low conceptual understanding of subject matter solely relied on procedures or algorithms in learning and did not develop different teaching strategies. Conversely, the teachers who understood both concepts and procedures were able to use innovative learning approaches. They could integrate pedagogical knowledge, context, and understanding of students with subject-matter knowledge. They could also develop appropriate learning strategies in the instructional topics. Similarly, Widjaja & Stacey (2009) showed that the teachers who were low in their understanding of concepts, relationships, and concrete models of fractions and decimals tended to teach the subject matter as mere arithmetic skills. In contrast, the teachers who have understood the meaning and concept of fractions and decimals could conceptually explain the subject matter to the students.

The results of the research showed that the instructional strategies used by teachers are not only influenced by the teacher's understanding of the subject matters, but the students' mathematical ability also affects their instructional strategies. The teachers who teach the students with middle or high math ability emphasize on the conceptual learning. They can find the procedural solutions in accordance with the rules or nature of the subject matter. Conversely, the teachers who teach the students with low math ability are difficult to implement a learning process conceptually. The students tend to understand the subject matter by remembering or memorizing. As a result, the teachers focus on giving exercises to the students in order to solve various types of problems.

The research results are strengthened by some previous studies of the differences in the students' ability and the teachers' instructional strategies. Shulman (1986) argued that understanding the students' concepts and pre-conceptions with different backgrounds and ages ask teachers to have the knowledge of determining appropriate instructional strategies so that students can understand a subject matter well. Furthermore, Park & Oliver (2008a) states that in the effective implementation of PCK, a teacher must understand the knowledge of students' conceptions of subject matter topics, learning difficulties, motivation, abilities, learning styles, interests, and needs. Van Garderen (2008) in his research of teacher learning practice in solving mathematics word problems states that a teacher needs to have knowledge of instructional strategies to teach problem solving to students who have learning difficulties. Some researchers have also pointed out that the use of different instructional strategies is due to the students with different cognitive abilities (Krawec, 2014; Mancl, Miller, & Kennedy, 2012; Garderen, Scheuermann, & Jackson, 2013; Garderen, 2008).

Conclusion

The teachers with the same educational background and teaching experience are similar to understanding the students' errors and difficulties in learning a subject matter. They can identify the errors commonly made by students in solving linear equations of one variable. They can explain the factors that lead the students to make them. The

students' mathematics ability is one of the factors that can influence the difference of instructional strategies used by teachers in teaching the solving of linear equations of one variable. The teachers who teach the students with high mathematics ability can implement conceptual learning that explains any solution by using principles or procedures that are in accordance with the rules. Conversely, the teachers who teach the students with low mathematics ability emphasize on giving exercises to solve various types of problems. The students prefer remembering or memorizing to understanding the procedure. Thus, it can be concluded that students' mathematical ability have an impact on instructional strategies used by teachers to teach to the students.

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